VOLUME V JANUARY, 1940 NUMBER 1

GEOPHYSICS

A Journal of General and Applied Geophysics

PROPAGATION OF ELASTIC WAVES IN THE EARTH*

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ABSTRACT

An investigation of elastic-wave propagation in near-surface materials using single-frequency continuous waves and pulses, over a range of 20 to 1400 cycles, is described. While only touching upon the diverse problems involved in a study of this kind the results indicate a complexity requiring considerable research if a proper understanding, commensurate with the importance of the problem, is to be attained.

INTRODUCTION

Although a large amount of work has been published concerning the propagation of elastic waves in the earth generated by both earthquakes and explosives, these investigations have, in general, been limited to the frequency band under 100 cycles per second and also to complicated wave trains in the main devoid of pure harmonic components. Thus, it seemed desirable to make a study with waves of single frequencies over not only this lower region but also in the upper range up to 2000 cycles per second. Of necessity, the evaluation of the relationships of frequency, attenuation, and velocity in the large variety of media occurring in the earth is a tremendous task; however, in the restricted work to be reported, some rather interesting results have appeared, both quantitative and qualitative.

APPARATUS

Since the conventional electromagnetic pickups are not suitable for recording very high frequencies, it was necessary to design a new type which responded to a wide range of frequencies. A satisfactory pickup was constructed using a cubical block of Rochelle salt crystal built up from four smaller crystals foiled on two faces. One face of the

* Paper presented at the Annual Meeting, Oklahoma City, March 21, 1939.

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cube was pressed firmly in contact with a diaphragm which was exposed to the water or mud which was placed in a hole drilled in the ground. A seismic wave applied a force on two opposite faces of the cube and generated an e.m.f. which was applied directly to the grid of a small tube mounted as close as convenient to the crystal. The output of the tube was coupled either directly or through a transformer to a long cable attached to an amplifier at the surface. Since the crystals must be kept dry for successful operation, a desiccant was generally inserted into the compartment with the crystal. This type of pickup proved sensitive to frequencies ranging from a few cycles to several kilocycles.

Two types of sources of ground vibration were used, an electrodynamic shaker and a magnetostriction tube. The electrodynamic shaker was constructed along the lines of an electrodynamic loud speaker. Various low resonance frequencies, mostly below 100 cycles, could be obtained by weighting the voice coil. The shaker could be operated with either the weight or the field magnet resting on the ground. It was found that very little energy was propagated through the dry surface layer at frequencies of several hundred cycles or more. Next a pipe extension with a driving plate on the end was made attached to the voice coil. When this driving plate was lowered in a hole penetrating the ground water layer, frequencies as high as 2000 cycles per second were transmitted with surprising amplitudes over short distances. It is interesting to note that, in the case of 20 cycle waves, large amplitudes were obtained when the shaker was applied to the surface of the earth, while on the other hand, small amplitudes were obtained when the shaker was applied on the bottom of a hole. This possibly indicates that a large amount of energy goes into surface waves of the Rayleigh type when the shaker is on the surface.

The shaker in its final form is shown in Fig. 1. The field magnet which is made of mild steel weighs about 300 pounds and is approximately 13 inches in diameter and 9 inches in height. The field winding consists of 459 turns of #8 copper wire and the voice coil consists of about 50 turns of #13 copper wire wound on a bakelite spool. The aluminum tube fastened to the voice coil is 12 feet long and 1.9 inches in diameter. The aluminum driving plate on the end is $3\frac{1}{2}$ inches in diameter. The shaker was usually handled with a tripod and chain hoist.

Various sources of power were used including the 60 cycle power line, a variable frequency oscillator coupled to a 100 watt amplifier, and a 500 watt 400 cycle generator driven with a gasoline engine. In

general, a bank of condensers of 100 microfarads capacity was used to neutralize the inductance of the 50 turn voice coil when excited with 400 cycle current.

The shaker system in air has a very strong resonance peak of the order of 400 cycles per second which seems to be the resonance frequency of the tube together with its slightly weighted ends. This resonance frequency is, of course, lowered when the driving plate is

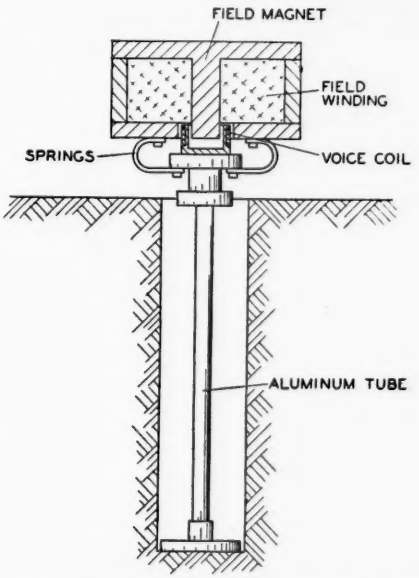

FIG. 1. Electrodynamic shaker resting in hole which as a rule contains water or mud at bottom.

in water and disappears when the full weight of the shaker rests on the bottom of the hole. A fairly large amplitude in the ground could be obtained at resonance with the driving tube suspended in water in a hole, but considerable loss occurred due to the turbulence caused by the large amplitude of the driving plate. Thus, although it is possible to absorb a large amount of motional power in this type of shaker, there is still a very important problem of coupling its motion to the ground.

AMPLITUDE MEASUREMENTS IN BORE HOLES

Since the energy distribution in different directions from the source is evidently far from uniform, it is of interest to determine the

decrease in amplitude of waves of various frequencies with distance from the source in at least two directions, vertical and horizontal.

The first of such measurements of amplitudes at different depths were made at the experimental station near the Pierce Junction salt dome using the electrodynamic shaker as the source. In this particular case, records which were made of pulses of various time durations indicated that after the short time (of the order of .03 seconds) required for the amplitude to build up there was little subsequent

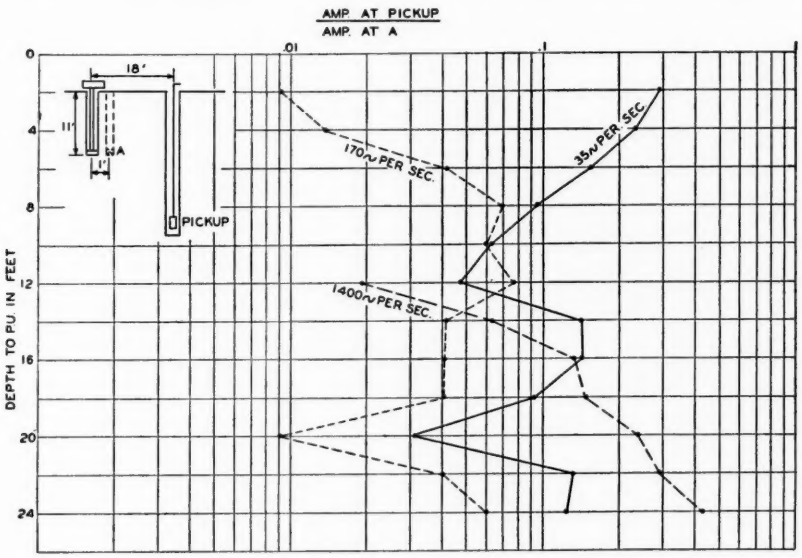

FIG. 2. Curves for various frequencies showing variation of amplitude with depth of pickup in configuration of shaker and pickup as shown.

change in amplitude such as might be due to energy arriving by a longer time path. Thus, in the case of these "vertical profiles," the direct wave was the only one observable on the records, and measurements might be equally well made with continuous waves as with short pulses. Since continuous waves were more convenient, they were employed in the measurements of Fig. 2, 3 and 4.

Fig. 2 shows the results obtained with the shaker set in an 11 foot hole and with the pickup lowered to various depths in a mud filled hole spaced 18 feet horizontally from the shaker. The amplitudes are all referred to the amplitude measured at a point spaced one foot horizontally from the driving plate. A very striking result is the

manner in which the amplitude near the surface decreases as the frequency increases whereas, at greater depths, the amplitudes of all frequencies become more nearly the same. In the case of 1400 cycles, there is strong evidence, as might be expected, of a strong downward directional effect.

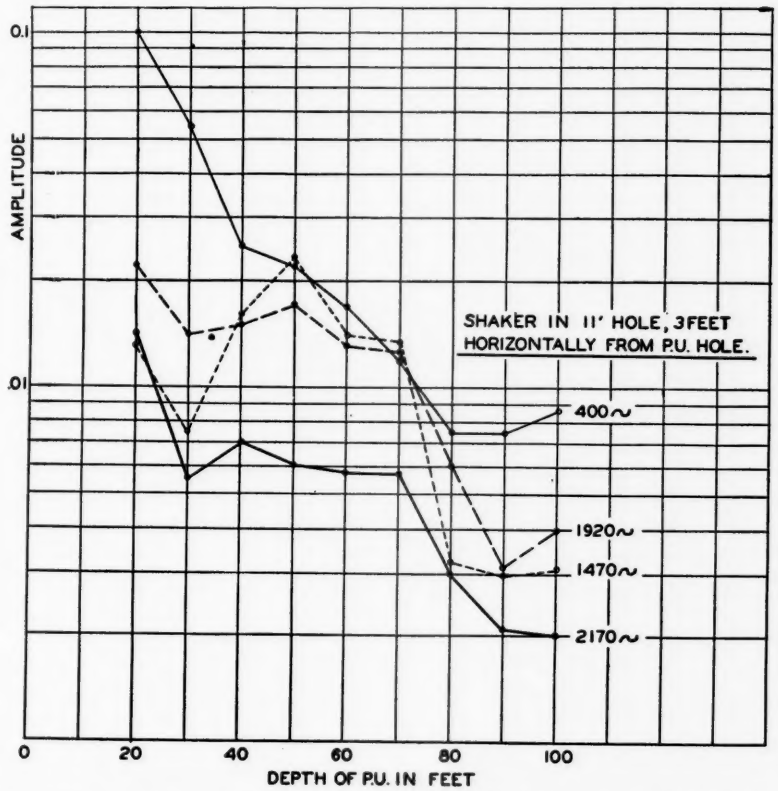

FIG. 3. Curves for various frequencies, showing variation of amplitude with depth of pickup the shaker being in a 11 ft. hole spaced 3 ft. from the pickup hole.

Fig. 3 shows amplitude measurements made at various depths down to 100 feet in a hole spaced 3 feet horizontally from the source. It was realized that, in this case, transmission down the mud column might be an important factor and hence a set-up was made as shown in Fig. 4. Here the driver was placed 50 feet horizontally from the pickup hole and measurements were made to a depth of 120 feet. The similarity of the results in these two cases indicate that the

transmission is through the earth and not down the mud column.

Three important points shown by these measurements should be mentioned. First, the amplitude is subject to wide variations over

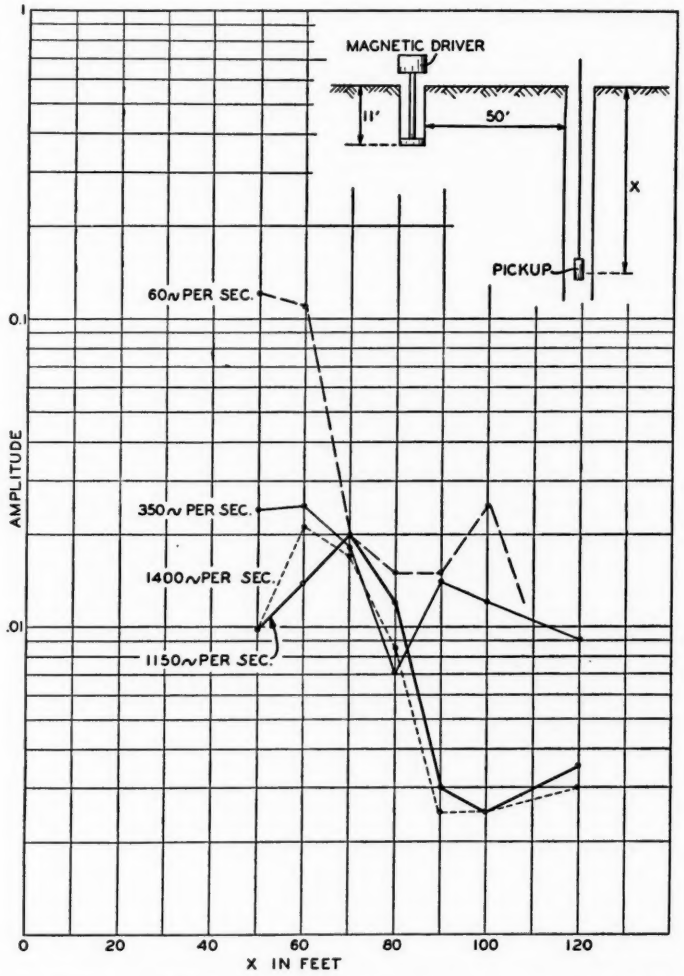

FIG. 4. Curves for various frequencies showing variation of amplitude with depth of pickup, the shaker being in a 11 ft. hole spaced 50 ft. from the pickup hole.

short distances, obviously due to effects other than attenuation and divergence. Second, the low frequencies seem to be propagated with a relatively higher amplitude near the surface but their amplitude

drops off very rapidly in the first ten or twenty feet below the surface. Third, at greater depths there is no evidence of a very great increase in attenuation of 1000 to 1500 cycle waves as compared to 60 cycle waves. The great irregularity in the amplitude curves, however, makes it impossible to determine attenuation factors with any significant

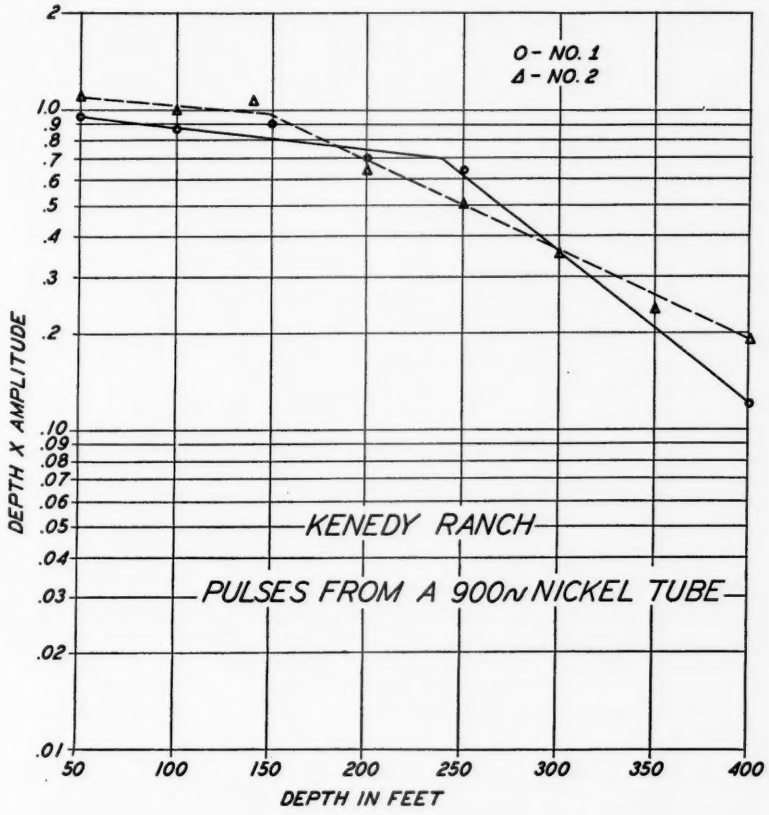

FIG. 5.

degree of accuracy, and it is evident that measurements over considerably greater distances are necessary if the attenuation is to be a dominant factor in affecting the amplitude.

Since it was impossible to keep a core hole of depth much greater than 100 feet open at Pierce Junction, the apparatus was adapted to portable use and taken to Kennedy County, Texas, where measurements were made in two 400 foot core holes.

A magnetostriction tube 8 feet long, 3 inches outside diameter and 1/10 inch thick was used as the source. It was driven by discharging a 20 m.f. condenser charged to 1100 volts through an 80 turn winding placed around the middle of the tube. This produced a highly damped pulse having a frequency of about 900 cycles per second. The tube was placed on end in a hole about five feet deep and spaced 50 feet horizontally from the core hole. Measurements on the amplitude of the pulse received at various pickup depths gave much smoother curves than those previously obtained, possibly indicating that the waves were traveling through a more nearly homogeneous medium than that at Pierce Junction. In any event, the curves afford a basis for estimating the magnitude of the attenuation in a vertical direction for the particular sections of earth involved.

In Fig. 5 the product of amplitude and depth is plotted against depth so that the attenuation factor may be determined directly from the slope of the curves as will be seen from the following consideration.

In a homogeneous medium the amplitude at a distance X from the source is given by

$$A = A_0 X^{-1} e^{-\alpha X}$$

where A_0 is the amplitude near the source, and α is the attenuation factor. This expression, of course, does not hold for very small values of X.

In the conventional manner, if the product AX is plotted on a logarithmic scale against X on a linear scale the result is a straight line having a slope $-\alpha$. This has been done in Fig. 5 where it appears that two values of attenuation were encountered in each set of measurements.

The values of α for X in thousands of feet and the distances required for the amplitude to be attenuated one half are given below.

Hole No.	*Depth*	α (*X in Kilo Feet*)	*Distance for Amplitude Decrease of One Half*
83	50–150	1.3	390 feet
83	150–400	9.0	78 feet
84	50–275	1.1	640 feet
84	275–400	6.5	108 feet

These values indicate a wide variation in the attenuation factor for different sections of earth and while it is true that the data given here leave much to be desired in the way of completeness, they furnish quantitative values of at least the right order of magnitude for the region involved.

One possible source of error in these measurements is a change in sensitivity of the piezo-electric pickup with pressure. Subsequent laboratory tests indicated an appreciable decrease in sensitivity (of the order of 20 per cent) for pressures of 200 pounds per square inch, which is roughly the pressure the pickup was subjected to at a depth of 400 feet. This means that the attenuation for the deeper beds are probably somewhat less that the values given in the above table.

PHASE VELOCITY MEASUREMENT

Phase velocity measurements for various frequencies were made at the experimental station at Pierce Junction by moving the piezo-electric pickup in a vertical hole (filled with mud) directly beneath the shaker. A cathode ray oscillograph was used to determine phase changes of standing waves of a given frequency. The amplifier output from the pickup was connected to the vertical plates of the oscillograph, and the oscillator output to the horizontal plates. As the pickup is moved in the hole, a pattern is obtained on the oscillograph which changes from ellipses to straight lines. The straight lines occur at intervals of $\frac{1}{2}$ wave length which can be measured with a tape line. Thus by measuring the wave length λ and the frequency γ, the phase velocity V can be calculated

$$V = \gamma\lambda.$$

Thus raising the pickup in a 30 foot hole and using 1900 cycles per second, 5 wave lengths were measured to be 13.7 feet. This gives a velocity

$$V = \frac{13.7}{5} \times 1900 = 5200 \text{ ft./sec.}$$

For 800 cycles, 1.5 λ measured 9′11″

$$V = \frac{9.9}{1.5} \times 800 = 5300 \text{ ft./sec.}$$

In a 100′ hole the following data were obtained:

1910 cycles/sec.	$7\lambda = 19.4'$	$V = 5300$ ft./sec.
420 cycles/sec.	$3\lambda = 40.3'$	$V = 5700$ ft./sec.

In these measurements the frequency and the distances were not determined very accurately. However, there is a qualitative agreement between the results and also with the results obtained with refraction

profiles to be described presently. With the results presented, there is a remarkable consistency in velocity at various depths and frequencies. It might be mentioned that some inconsistent and strange results were obtained at shallow depths and also with low frequencies. The method offers possibilities for making accurate determinations of phase velocities at frequencies sufficiently high to give a number of wave lengths of path in a homogeneous bed.

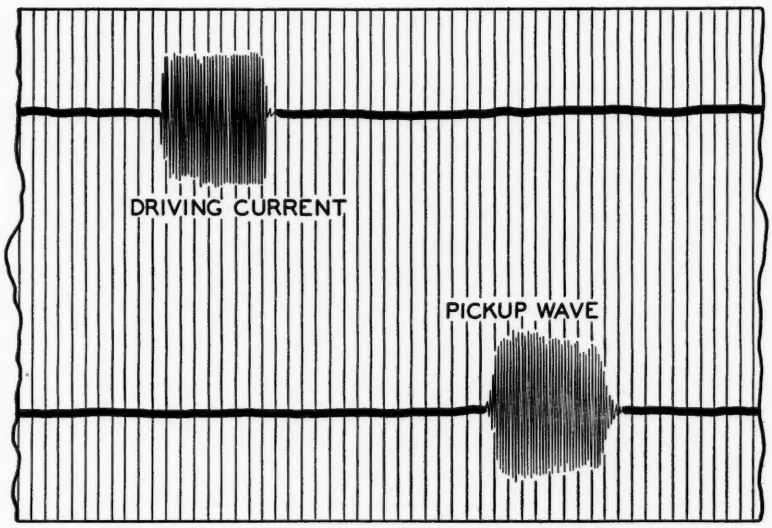

FIG. 6. Tracing of an oscillogram taken with pickup at a distance of 1200 ft. from shaker which was driven with 400 cycle current. The vertical time lines are spaced with an interval of 0.01 sec.

REFRACTION PROFILES

To obtain information on the horizontal transmission of high frequency seismic waves, a number of horizontal refraction profiles were run using high frequency pulses of short time duration. The transmission times were accurately measured, and observations were also made regarding the received amplitudes. The first of these profiles was run on top of the Hawkinsville salt dome along a line where the depth of the cap rock varied from about 100 to 200 feet. The source was the electrodynamic shaker which was set in a 10 foot hole partly filled with water. Copper wire fuses were used to interrupt the 400 cycle driving current supplied to the shaker so that wave trains a few

hundredths of a second long were obtained. The pulses were recorded by means of the piezo-electric pickup which was placed at the bottom of a 10 foot hole and covered with water. A wire line was strung from the source to the recorder so that an oscillogram of the driving current supplied to the shaker could be recorded along with the pickup wave. Thus, the transmission time could be read off directly. Fig. 6

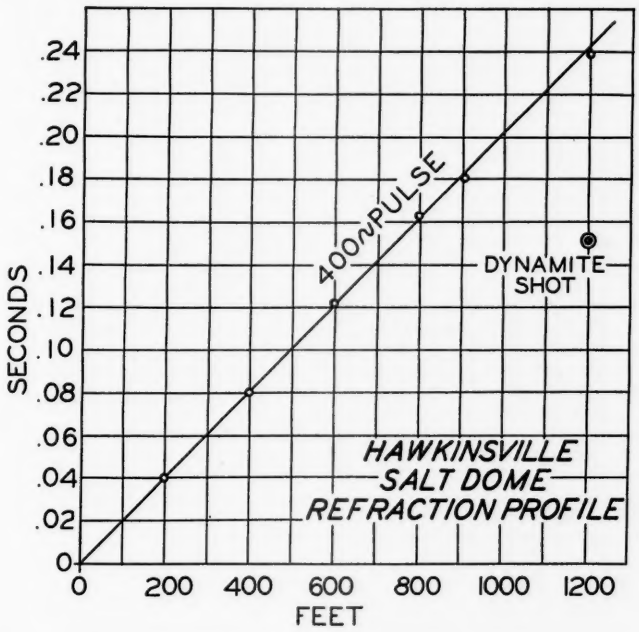

FIG. 7. Refraction profile made over Hawkinsville Salt Dome, Texas.

shows a typical record, the distance in this case being 1200 feet. It is to be noted that the pickup wave resembles very closely the expected transmitted wave as may be seen from the driving current wave after allowance is made for the shaker characteristic.

The travel-time curve obtained is shown in the accompanying Fig. 7; it is a straight line through the origin. The velocity obtained from the slope is 5000 feet per second. It will be noticed that a dynamite explosion is recorded with a much smaller travel-time at 1200 feet than that of the 400 cycle wave. This would indicate that the dynamite first kick is from a pulse through the cap rock while the 400 cycle train probably travels through the upper layer, any 400 cycle

energy which went through the cap rock being too small to be detected.

In this connection, it should be mentioned that the noise level (not shown on the tracing) on the original record from which Fig. 6 was traced was about one-tenth the amplitude of the pulse. Thus, no waves having less than one-tenth the amplitude of the pulse could have been observed. Further, the location of the shaker appeared to

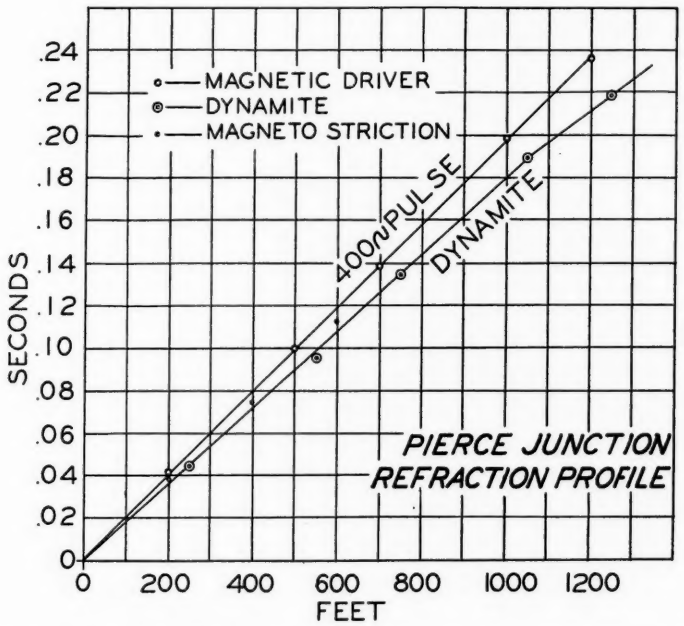

FIG. 8. Refraction profile made at Pierce Junction, Texas, experimental station with 400 cycle train from electrodynamic shaker, with 900 cycle damped wave train from magnetostriction tube, and with dynamite pulses.

be an important factor in determining the amplitudes. At one location only 100 feet from that used in making the measurements, no energy could be detected at a distance of 1000 feet. The shaker, in this case, was set at the edge of a bayou. Also with the set-up used in obtaining the curve, the amplitude of the wave train at 1200 feet (see Fig. 6) was comparable with the amplitude at 200 feet. This would indicate that the 400 cycle energy was probably confined to a large extent in a layer.

A similar refraction profile was run near the Pierce Junction salt

dome with results as shown in Fig. 8. More points with dynamite shots were obtained and in addition a profile was run with damped waves obtained from a magnetostriction tube having a frequency of approximately 1000 cycles per second. The velocity for the 400 cycle wave trains is about 5100 feet per second, very close to the value obtained at Hawkinsville. A somewhat higher value is obtained for the dynamite pulses, the discrepancies not being so great as at Hawkinsville. The damped wave points seem to fall in between the curves, indicating that shorter times are read off when large first fronts are used. It might be mentioned that a wave train of 1500 cycles per second obtained with the electrodynamic shaker gave very closely the same time at 200 feet distance as the 400 cycle train.

SUMMARY

An investigation has been undertaken concerning the propagation through the earth of seismic energy in the form of controlled, single frequency wave trains and pulses. One important aspect of such an investigation lies in the fact that a knowledge of the frequency characteristics of the earth in transmitting seismic waves provides a basis for determining the nature of the disturbance at any point in the earth resulting from any type of impulses whatever. Thus, the information gained has a direct bearing on seismic exploration methods as used at present as well as on any proposed new methods. The results indicate that the problem is a very complex one and that the work herein described does little more than scratch the surface.

In general, the seismic characteristics of the earth have been found to vary widely with frequency. In dry surface layers the attenuation to high frequencies is very great, but it drops to much lower values in the deeper, water saturated beds. On the other hand, low frequencies of 20 to 60 or even 100 cycles per second are not seriously attenuated in the surface layers; but there is evidence that the attenuation in the deeper beds is nearly as great and in some instances even greater than for the high frequencies. An example of this is the relatively higher amplitude of a 400 cycle pulse as compared to a 60 cycle pulse after being transmitted 1200 feet in a horizontal direction at Pierce Junction. In all this, of course, it must be pointed out that all measurements have been made over relatively short distances and that no accurate determinations of attenuation factors have been possible.

In the case of horizontal refraction profiles using high frequency

pulses, all velocities observed have been remarkably close to the velocity of sound in water. The travel-time-curves show no break; that is, they are straight lines through the origin and do not indicate any higher velocities picked up at distances up to 1200 feet. Furthermore, the amplitude does not fall off with distance as fast as might be expected from vertical transmission data. In one instance, the decrease in amplitude was considerably less than according to the inverse distance law.

One possible interpretation of this is that the high frequencies are more or less confined to certain strata which would be expected to have good reflecting boundaries. However, attempts to observe such reflections at suitable distances from the source have yielded no positive results. On the other hand in some cases, the maxima and minima observed in amplitude measurements in bore holes give a strong indication that reflection or refraction interference patterns are of considerable importance at least in the shallow beds.

The principal problems in extending this investigation to cover greater distances and so include a wider variety of sections of earth are concerned with the construction of more powerful sources, and with the development of methods of coupling vibrating systems to the earth so as to get a maximum amount of energy transferred into seismic waves. In conclusion it is felt that, although the problem is far from solved, the results of this paper are encouraging and it is hoped that further researches will be stimulated.

VELOCITY STRATIFICATION AS AN AID TO CORRELATION*

ROLAND F. BEERS†

ABSTRACT

This paper outlines the historical development of the concept "velocity stratification" from the field data of refraction and reflection seismographs. The concept is shown to form the basis of the phenomenon known as "seismic reflections." Experimental data are interpreted to show approximately the magnitude of reflection coefficients. Employment of velocity stratification as a means of stratigraphic correlation is suggested. Means whereby data for these correlations may be collected are discussed. Finally the use of the method for attack on stratigraphic problems is outlined.

The differentiation of sedimentary beds by seismic velocities was indicated as a possibility in the development of a technique leading to structural delineation by means of the refraction seismograph in the Permian Basin of west Texas in 1928. It was expected that masses of Permian salt would exhibit velocities of propagation of seismic waves greater than those of the clastic sediments of the same age. This anticipation was the outgrowth of experience in the Texas and Louisiana Gulf Coasts where intrusive salt masses showed a longitudinal wave velocity of 15,000 to 16,000 feet per second in contrast with velocities of 6,000–7,000 feet per second for surrounding sediments.[1,2]

An unexpected result of the application of the refraction method to Permian stratigraphy was the discovery that other sedimentary beds of Triassic to Permian ages exhibited marked stratification in velocities. When this discovery had been confirmed by sufficient experience, it became possible to establish correlations between various seismic stations solely on the basis of the seismic wave velocities. These velocities were those of horizontal beds comprising a greater portion of the path of travel of these waves. When the time-distance graphs of refraction profiles were plotted it was found that an immediate identification of the various horizontal beds was available from these propagation velocities. The clarity with which the Triassic and Permian stratigraphy was delineated by this method became so outstanding that little question was raised regarding the basis of such

* Presented at the Annual Meeting, March 21, 1939.

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[1] "Seismic Method of Mapping Geologic Structures" by Donald C. Barton, Geophysical Prospecting, A.I.M.M.E., 1929.

[2] E. DeGolyer, Journal, Society of Petroleum Geophysics, Vol. VI, No. 1, p. 5.

stratification. Fig. 1 shows an example of the velocities measured by this means in Ector County, Texas.

Since the development of the reflection seismograph, additional evidence has been gathered on the phenomenon of velocity stratification in this same province. It is now clear that stratification accord-

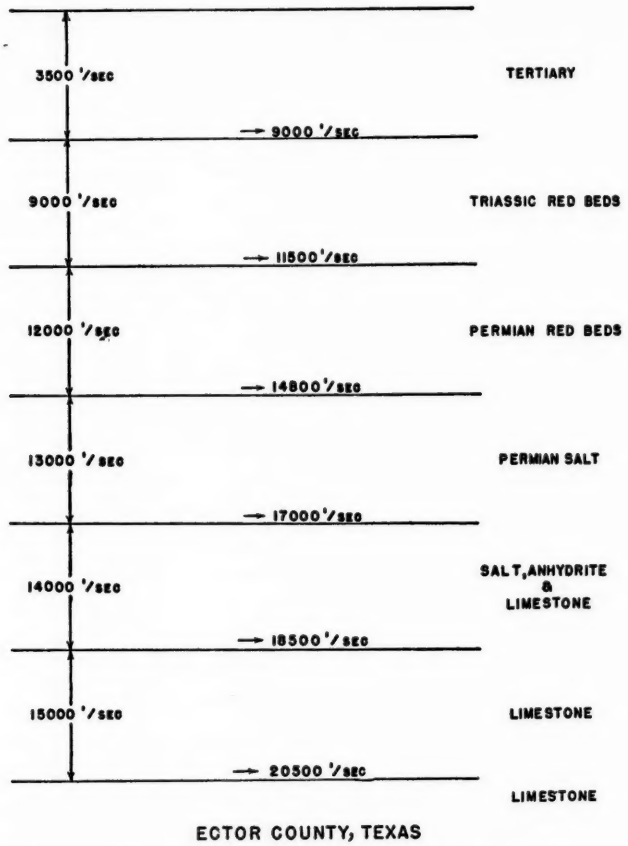

FIG. 1. Schematic diagram showing type of velocity stratification, Ector County, Texas.

ing to horizontal velocities of propagation is somewhat more pronounced than stratification by vertical velocities. This observation adequately accounts for the success attained by the refraction seismograph in this province and establishes more firmly the basis upon which the observations were computed. It was recognized early in

the application of the method that some advantage would accrue if depths to the various refracting horizons were computed on a layer-to-layer basis. In other words, if the differential velocity exhibited in the vertical travel of a seismic wave through each type of formation was determined, then the travel times through each of these lithologic units could be used to calculate the thickness of these units. Development of this concept led naturally to the idea of vertical velocity stratification. This characteristic was also readily recognized when velocity measurements were made in open bore holes by seismic means.

A further advantage of the plan of breaking up the entire geologic column into rational lithologic units is the consequence that the accuracy of the depth calculations to deeper beds is enhanced by this method. The value of this plan persists in the application of well shooting data to reflection seismograph surveys.

The phenomenon of seismic reflections gives substantial basis to the concept of velocity stratification in sedimentary beds. Rayleigh,[3] as well as others,[4] has shown that an actual discontinuity in acoustic properties must exist in order that reflections of recognizable amplitude shall be produced at the interface of two such beds. According to Rayleigh, a sudden change in compressibility or in density will satisfy the requirements for the reflection. The conditions are expressed more generally in the relationship between the acoustic impedances of the two media, as first expressed by Webster,[5] in more concise form by Stewart and Lindsay.[4] The ratio of the amplitude of the reflected wave to that of the incident wave, at normal incidence is given by:

$$B/A = (R_2 - R_1)/(R_2 + R_1),$$

for longitudinal waves, where

B = amplitude of reflected wave in medium 1
A = amplitude of incident wave in medium 1
R_1 = specific acoustic impedance of medium 1
R_2 = specific acoustic impedance of medium 2
$R = \rho V$
ρ = density of medium
V = velocity of longitudinal waves

[3] Rayleigh, "Theory of Sound," Volume 2, paragraph 270.

[4] Steward and Lindsay, "Acoustics," paragraph 4. 2.

[5] Webster, A. G., Proc. Nat. Academy Sci., 5, 275, 1919; H. Brillié, LeGénie Civil, 1919.

In most cases, changes in the values of acoustic impedance are brought about by changes in the velocity of propagation of longitudinal waves. This condition is true at least in the case of most clastic sediments examined in the course of the present experiments. On this basis, the value of the reflection coefficient reduces to the expression:

$$B/A = (V_2 - V_1)/(V_2 + V_1)$$

where V_1 = the velocity in the incidence medium
V_2 = the velocity in the refracting medium

From the data of Fig. 1, the theoretical reflection coefficient existing at various lithologic contacts in the Permian Basin has been computed. Since the velocities of horizontal propagation are not equal to the velocities of vertical propagation, these reflection coefficients are probably in excess of those actually existing. Experimental evidence has shown that many sedimentary formations are not isotropic and that the horizontal velocity of a formation may exceed the vertical velocity by 10%–20%. Table 1 shows the coefficients computed from the ratios of vertical to horizontal velocity at each interface.*

TABLE 1

Velocity in Incidence Medium V_1	*Velocity in Reflection Medium* V	*Reflection Coefficient* $\frac{V-V_1}{V+V_1}$
9,000	11,500	.122
12,000	14,800	.104
13,000	17,000	.133
14,000	18,500	.138
15,000	20,500	.155

Confirmation of the idea of velocity stratification is found in the many examples of seismic data gathered by well shooting. When a detector is lowered into a bore hole, the time of travel from the surface to the detector at various positions can be analyzed to show that the velocity of propagation of longitudinal waves varies according to the lithologic properties of the formation encountered. Since most well

* While a rigorous treatment of elastic-wave propagation in anisotropic media is not available at present, the method of calculating these reflection coefficients using ratios based on vertical velocity contrasts would be more orthodox, and applicable under conditions of normal incidence of plane waves when longitudinal-transverse wave transformations could be ignored.—Ed.

shooting data taken up to the present time have not been gathered with this concept in view, there is unfortunately little evidence available for close study of these factors. With an appreciation of the values to be obtained by such studies it is to be expected that future well shooting will be taken at more closely spaced positions of the well detector and that means will be developed whereby greater resolution of the velocity characteristics of formations will be had. It is recognized that large amounts of these data have been gathered by independent organizations which might be of great value for study and correlation if they could be made available to some disinterested agency. In the absence of access to these data it has been necessary to proceed toward the gathering of such evidence as could be had by experimental means. With limited facilities enough evidence has been gathered to show that correlation of lithologic units can be established over considerable distances by means of measurements of the individual velocities of these units. It is probably true that the differentiation of beds by this means is more striking than by any other physical method in present use. This occurs because of the wide range which various beds exhibit in the magnitude of velocity of propagation. Importance of this statement can be recognized when it is noted that seismic velocities in sedimentary beds range from 2,000 to 24,000 feet per second. Clearly, there are few physical quantities yet observed in connection with properties of rock *in situ* which offer such a wide range of points of differentiation. A close study of the velocities and correlation with lithologic properties leads to the conclusion that a new geologic tool of great resolution has become available.

The means by which data may be gathered for this purpose are many. The most obvious plan would be to lower a detector into a well at closely spaced positions and to record the time of travel of seismic waves from the surface. When this method is carried to the limit of geologic resolution it is found that the accuracy is insufficient. In this scheme the differential velocity of various beds is determined by the difference in time of travel of the seismic waves to the top of a bed and to its lower face. Since this time difference may be of the order of 10^{-4} seconds it is seen that prevailing methods of time measurement will be inadequate.

A second scheme has been tried which consists of lowering a sound generator into the hole at a spaced position from a detector. By varying the frequency of the sound generator it is possible to produce an interference pattern between the generator and the receiver and

thereby to determine the velocity of propagation of the lithologic unit embraced between the generator and the detector. The precision of the measurement obtained by this method is satisfactory but dif-

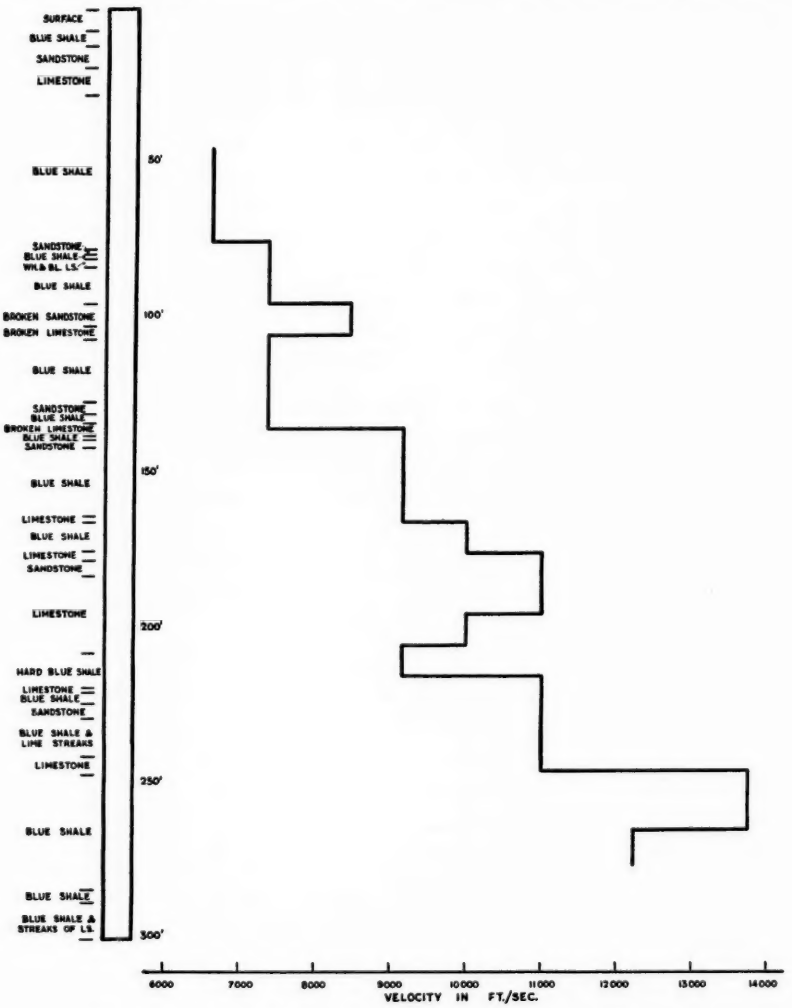

FIG. 2. Differential velocities measured in Comanchean section, Tarrant County, Texas.

ficulty will be encountered from standing waves in the bore hole. These are produced by reflections from the surface of the fluid in the hole and at other points.

Salvatori has shown a method which embraces the use of two detectors at a spaced interval. These are lowered together into the bore hole and serve to record the instant of arrival of a seismic wave which is generated at the surface of the ground. This method is covered by U. S. Patent No. 2,137,985.

Other methods are undoubtedly in development which will serve to increase the accuracy of the measurement of time required in this connection. By methods which are not now available for publication, data shown in Fig. 2 have been gathered.

The results of these investigations have shown that correlation by means of velocity stratification can be accomplished and often affords the clue to obscure stratigraphic relationships which are not otherwise clear. The experiments have led to a closer view of the origin of reflections. When accurate velocity data have been available, they have shown that the actual identity of the reflecting beds can be determined with certainty. The appearence of reflections hitherto not understood has been explained. It has also been observed that reflections may occur at the interface of two formations having sufficiently different values of velocity, irrespective of whether the reflecting bed has a smaller or larger velocity than that of the incident bed. Refinements of this method will eventually show the limit of resolution of the reflection seismograph. Since stratigraphic problems are becoming more intricate it seems advisable to determine how these problems may be best attacked. If the resolution of the reflection seismograph can be determined by design and if the requirements are well defined, then perhaps a better attack on these problems may be prepared.

A further result of this work has been a more intimate understanding of the entire reflection path of travel. Rayleigh has shown certain conditions can exist where no reflections occur. These frequently depend upon velocity and spread relationships. If these relationships can be determined by closer study of velocity considerations, solutions may be forthcoming which were hitherto unavailable.

Experience gathered by shooting wells along the plan suggested will support or destroy the contention that velocity logs of this type offer new and valuable means of correlation and that they may constitute, by use of the reflection seismograph an aid to an attack on the problem of stratigraphic traps.

AN EXPERIMENTAL DEVICE FOR COMPUTING MAGNETIC AND GRAVITATIONAL ANOMALIES*

J. W. FISHER, Ph.D.†

ABSTRACT

Based upon the fact that gravitational and magnetic anomalies are functions of the solid angle subtended at the point of measurement by the contours of the anomalous body, an electro-magnetic model is described which permits direct measurement of the desired quantities in terms of mutual inductance. A model of the subsurface "structure" is made in the form of a coil in which the vertical spacing of the turns is kept constant and the shape of the helix conforms to the contours of the structure. A small search coil is then located at the point corresponding to that at which a gravimetric or magnetometric observation is desired. The mutual inductance between the search coil and the main coil is then measured and may be expressed in terms of the magnetic or gravity anomaly arising from the simulated structure. For magnetic anomalies the search coil is made as compact as possible whereas in the case of gravity anomalies the search coil is in the form of a long solenoid of small diameter, evenly wound along its length, in order to affect the integration necessary to measure the magnetic potential analogous to the desired gravitational quantity.

An important section of Geophysics as applied to prospecting is based on measurements of the small changes in gravitational or magnetic force produced by differing geological formations within some given region. In gravitational surveys, variations in the vertical gravitational acceleration "g" may be measured directly by some type of static gravimeter, while the various second derivatives of the gravitational potential U may be determined by a torsion balance. Of the different magnetic elements which may be measured, the changes in the vertical magnetic field V are the easiest to measure and are also easiest to interpret. The interpretation of such surveys may offer many difficulties, even when the problem is idealized by assuming that the structures causing the anomalies are composed of materials homogeneous in density and magnetic susceptibility and bounded by a single differential surface. In what follows an account is given of some theoretical considerations bearing on this question and an experimental electromagnetic method is described, which has been devised for the rapid and accurate determination of certain anomalies to be expected from a given hypothetical structure, which would otherwise demand days of laborious work using graphical methods of integration. Even in problems which can be treated as 2-dimensional, the calculation of the anomalies due to comparatively simple

* Manuscript received Sept. 8, 1939.

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structures may be very laborious, while the converse problem of deriving a hypothetical structure from an observed anomaly can be solved only by trial and error; it is hoped that the experimental methods proposed will at any rate speed up and simplify the former.

The present line of investigation owes its origin to a neat theoretical simplification due to Dr. J. H. Jones, who noticed that the gravitational and magnetic anomalies, due to a thin horizontal slab of material bounded by a given contour, could be expressed very simply in terms of the solid angle subtended by the contour at the point where the anomaly is measured. Suppose that the rock mass is divided up by horizontal planes into slabs of vertical height Δz. Let C be the contour of such a slab at vertical depth z below a point P on the ground surface, and let ω be the solid angle subtended at P by C. Further let ρ denote the relative density of the rock and k its relative susceptibility. The vertical gravitational anomaly at P due to this slab contour will be,

$$\Delta g_z = G\rho\omega\Delta z \tag{1}$$

where G is the universal constant of gravity. The vertical magnetic anomaly at P, produced by induction of the slab in the earth's magnetic field H, is expressed by,

$$\Delta H_z = k\left(H_x\frac{\partial\omega}{\partial x} + H_y\frac{\partial\omega}{\partial y} + H_z\frac{\partial\omega}{\partial z}\right)$$

or, in vector notation

$$\Delta H_z = k(\boldsymbol{H}, \text{grad } \omega)\Delta z. \tag{2}$$

Thus the vertical gravitational anomaly depends on the solid angle ω, while the vertical magnetic anomaly depends on the gradient of this solid angle. There is in fact a general relation between the total gravitational and magnetic anomalies produced by the same homogeneous rock mass, viz.,[1]

$$\delta\boldsymbol{H} = \frac{1}{G}\,\frac{k}{\rho}\,(\boldsymbol{H}\,\text{grad})\delta\boldsymbol{g} \tag{3}$$

in which $\delta\boldsymbol{H}$ and $\delta\boldsymbol{g}$ represent the vector magnetic and gravitational anomalies respectively. We can write (3) a little differently if we introduce a scalar length h measured in the direction of $\boldsymbol{H}$. Then

[1] The relation between δH and δg was recognized by Eötvos; see for example Gutenberg "Lehrbuch der Geophysik" pp. 557–560.

$$(\boldsymbol{H}\ \text{grad})\delta\boldsymbol{g} = |\,H\,|\frac{\partial}{\partial h}\delta\boldsymbol{g} = \frac{1}{r}\delta\boldsymbol{H} \quad \text{if} \quad r = \frac{1}{G}\frac{k}{\rho}\cdot \tag{4}$$

The direction of $d\delta\boldsymbol{g}$ will, therefore, be that of $\delta\boldsymbol{H}$, where $d\delta\boldsymbol{g}$ is the infinitesimal change in $\delta\boldsymbol{g}$ for a displacement dh in the direction of $\boldsymbol{H}$. We could introduce the potential U of the gravitational anomaly by writing,

$$\delta\boldsymbol{H} = r\,|\,H\,|\,(\text{grad}_h)\ \text{grad}\ U. \tag{4a}$$

If we restrict ourselves to the 2-dimensional case with $\partial/\partial y = 0$ and assume also $H_x = 0$, i.e. that the magnetic meridian lies along the y axis, we get the following very simple relation,

$$\delta\boldsymbol{H} = r\,|\,H\,|\frac{\partial}{\partial h}(\delta g_x + \delta g_y + \delta g_z) \tag{5}$$

but $\delta g_y = 0$, since the problem is now 2-dimensional and

$$\frac{\partial}{\partial h} = \frac{\partial}{\partial x}\frac{dx}{dh} + \frac{\partial}{\partial z}\frac{dz}{dh} = \sin D\frac{\partial}{\partial z},$$

since

$$\frac{\partial}{\partial y} = 0; \qquad \frac{dx}{dh} = 0.$$

D is the angle of dip. Thus the vector equation (5) reduces to

$$\delta\boldsymbol{H} = r\,|\,H\,|\,\sin D\frac{\partial}{\partial z}(\delta g_x + \delta g_z),$$

so that,

$$\delta H_x = r\,|\,H\,|\,\sin D\frac{\partial^2 U}{\partial x \partial z}; \qquad H_z = r\,|\,H\,|\,\sin D\frac{\partial^2 U}{\partial z^2}\cdot \tag{6}$$

Incidentally $(\partial/\partial x)\delta H_z = (\partial/\partial_z)\delta H_x$, since curl $\delta\boldsymbol{H} = 0$ in a static field. The 3-dimensional interpretation of vertical magnetic anomalies may be based on Eq. (3). The y axis may always be chosen so that $H_y = 0$, and then for any given contour C, the values of ω, $\partial\omega/\partial x$, $\partial\omega/\partial z$ can be found for a point P, at a given distance away, by means of suitably constructed graticules or charts on transparent paper, constructed for a given value of z. 3-dimensional charts for magnetic anomalies have been worked out by S. J. Pirson,[2] but their form, al-

[2] "Polar charts for Interpreting Magnetic Anomalies" S. J. Pirson. American Institute of Mining and Metallurgical Engineers. Contribution No. 91, Feb. 1936.

though applicable to a structure of any shape and depth, does not involve the solid angle or its gradient directly, and so does not suggest the electromagnetic analogy, mentioned later, which follows so simply from the form suggested here. The center of the chart is placed over the point P' at the foot of the perpendicular from P to the plane of C and the number of chart segments enclosed by C is noted. The ω chart (Eq. 1) for the gravitational problem is the simplest, since it is of circular symmetry and every segment corresponds to a unit positive contribution. For the magnetic problem we require a $\partial\omega/\partial z$ chart for the vertical component of the earth's field and a $\partial\omega/\partial x$ chart for the horizontal component, supposing that the latter is directed entirely along the x axis. Both of these gradient charts have segments with positive and negative unit contributions. Theoretically the anomaly

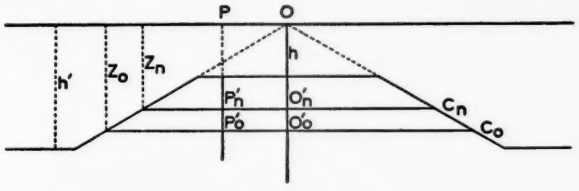

FIG. 1.

produced by any rock mass could be estimated by dividing it up into an appropriate number of slab contours and then applying charts in turn to these. This, however, at once raises two complications for, in the first place, we should require a different chart for each depth of contour, and secondly, we should have to determine corresponding points P′ on each contour corresponding to the same surface point P. There is, however, one case in which it is fairly easy to deduce the effect of the whole rock mass from chart measurements carried out over one contour only. This can be done when all the slab contours have the same shape but with linear dimensions proportional to their depths below the surface, in other words, when the mass forms a truncated conical mass with its apex in the surface. Let C_0 be the horizontal cross-section contour corresponding to the depth z_0 for which the chart is constructed. Consider first the solid angle chart, the linear dimensions of each segmental unit of which will be proportional to the depth z for which it is intended. The effect of a contour C_n at a depth z_n at a surface point P is found by using a chart for depth z_n applied over the contour C_n with its center at

P_n', the point vertically beneath P. Let O_n' be the center of the contour C_n. The dimensions of the standard chart for depth z_0 will be $a_n = z_0/z_n$ times those for a chart for depth z_n, so that we should get the same result by applying the standard chart to a contour of dimensions a_n times those of C_n but of similar shape. The distance from the center of this reduced contour to the center of the standard chart will also be a_n times its value for the contour C_n or say $a_n r$, if $r = P_n'O_n' = PO = P'O'$. Therefore, in order to find the sum total of the effects of all the slab contours at a given surface point P at a distance r from the fixed point O, where the vertical axis through the centers of the sections meets the surface, we have to sum the readings S on the standard chart for all distances $a_n r$ from its center giving, since $S = S(a_n r)$, for the total vertical gravity anomaly

$$\delta g(r) = k \sum S(a_n r) \Delta z_n; \qquad k = G\rho.$$

Increasing the number of contour cross-sections indefinitely over a vertical range of z from h' to h, we can write

$$\delta g(r) = k \int_h^{h'} S(a_n r) dz, \quad \text{or if} \quad \eta = a_n r = \frac{z_0}{z_n} r$$

$$\delta g(r) = k r z_0 \int_{\eta''}^{\eta'} \frac{1}{\eta^2} S(\eta) d\eta; \qquad \eta' = \frac{z_0}{h'} r, \quad \eta'' = \frac{z_0}{h''} r.$$

This integral can be found graphically by plotting corresponding values of $(1/\eta^2)S(\eta)$ and η and then, by choosing different values of r, the surface distribution of the vertical gravity anomaly about O can be estimated. The $S(\eta)$ will of course depend also on the azimuth ϕ, but corresponding chart points for all contours have the same value of ϕ. The $(1/\eta^2)S(\eta).\eta$ graph must, therefore, be drawn for various azimuths. As already mentioned this method is applicable only to masses forming a truncated cone with apex in the surface plane, as it is in this case only that all reduced contours for the standard depth z_0 coincide.

In applying similar ideas to the solid angle gradient charts, we must take into account the fact that changing the depth at which a chart applies involves also a change in value of the unit of gradient measured by each chart segment. In fact the product of the scale of the chart and magnitude of unit is invariant. Thus in this case, taking as an example the vertical magnetic anomaly due to the vertical component H_z of the earth's magnetic field we have,

$$\delta H_z(r) = k H_z \sum a_n S(a_n r) \Delta z_n$$

or, proceeding to the integral form,

$$\delta H_z(r) = kH_z \int_{\eta''}^{\eta'} \frac{1}{\eta} S(\eta) d\eta.$$

The same will apply to the $\partial\omega/\partial x$ chart giving the contribution to the vertical magnetic anomaly produced by the horizontal component of the earth's field.

If h, h' denote the depths to the top and bottom surfaces respectively of the truncated cone, and if we choose $z_0 = 1$, we must put $\eta' = r/h'$ and $\eta'' = r/h''$. For surface points near O for which $r << 1$, we can put $S(\eta) = S_0$, where S_0 is the reading for the center of the standard contour. We get then for the anomaly over the center of the mass

$$\frac{\delta H_z}{kH_z} = S_0 \log h'/h.$$

Since the $\partial\omega/\partial z$ chart contains a negative central zone and outside this a positive zone, there will clearly be a maximum possible value of S_0, namely that for a circular contour C_0 coinciding with the circular boundary separating the two regions of the chart. The maximum value is

$$\frac{4\pi}{3\sqrt{3}} \log h'/h = 2.42 \log h'/h.$$

The above method of chart integration can be extended with slight modification to the case of a rock mass in the form of an oblique truncated cone with its apex in the surface.

EXPERIMENTAL METHOD OF COMPUTATION

We have seen that the vertical magnetic anomaly due to a flat contour slab of thickness Δz magnetized by the earth's field H is,

$$\delta H_z = k\Delta z(\boldsymbol{H}, \text{grad } \omega),$$

ω being the solid angle subtended by the slab at the point of measurement. Now if the contour were bounded by a circuit carrying a current i, the magnetic effect of the circuit would be equivalent to that of a magnetic shell of strength i. Thus the magnetic potential at a point P would be given by

$$\Omega = 2\pi ni = \omega i, \quad n \text{ being any integer.}$$

The magnetic field $\boldsymbol{h}$ at P due to the circuit will be given by the gradient of Ω, so that

$$\boldsymbol{h} = \text{grad } \Omega = i \text{ grad } \omega.$$

Thus we can write

$$\delta H_z = \Delta z \frac{k}{i} (\boldsymbol{H}, \boldsymbol{h}) = c \left| \boldsymbol{H} \right| h_H$$

where h_H is the component of the vector $\boldsymbol{h}$ in the direction of $\boldsymbol{H}$ and C is a constant. The product on the right can be measured experimentally, without hindrance by the earth's field, if we employ an alternating current $i = i_0 e^{2\pi i v t}$ in the boundary circuit. The required component h_H can then be sorted out by using a small search coil,

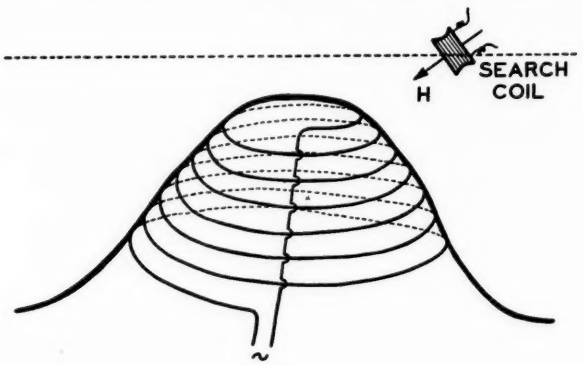

FIG. 2.

placed in relation to the boundary circuit so that its plane would be perpendicular to the direction representing that of the resultant magnetic field $\boldsymbol{H}$ of the earth or, in other words, by making the axis of the search coil point along this representative direction. We could then measure the induced E.M.F. in this small coil or, what turns out to be more convenient in practice, the mutual inductance between it and the boundary circuit.[3] The values of this for any positions of the small coil with respect to the boundary circuit, always keeping the axis of the search coil in the right direction, will give the relative values at these points of the vertical magnetic anomaly produced by the slab under the combined action of both vertical and horizontal components of the earth's field. To proceed from the contour slab to the whole mass is an obvious step, for all we have to do is to add a

[3] The measurement of the mutual inductance with the Campbell inductometer bridge was suggested by Mr. W. L. Beck of the Cambridge Instrument Company.

new circuit carrying the same current for each contour section or, what comes to the same thing, wind a coil round a form representing the structure, keeping the vertical spacing of the turns constant, and pass an alternating current through the whole coil. In this way we can deal with structures of any shape.

The method has been tried out using rectangular and elliptical contours, on a scale of $4''=1$ mile, with a small coil of 1,500 turns as search coil. The mutual inductance between the two coils was measured by using a Campbell inductometer bridge supplied by an oscillator generating at 800 cycles per second. The mutual inductance is read off directly on the bridge scale by moving the recording arm until no sound is heard in the telephone. By using in addition an amplifier and a small balancing rheostat to control the phase relations in the bridge circuits, very precise settings are possible. The results obtained for the above contours agreed extremely well with those obtained approximately by days of work using the charts as explained, and each axis, corresponding to a given azimuth, took but a few minutes to complete. As a precision method for the investigation of any given structure this inductometer method would appear ideal. Its use moreover is not necessarily restricted to the solution of magnetic problems, but may also be employed for the interpretation of gravitational anomalies as determined by a gravimeter and the second derivatives of the gravitational potential U as determined by the Eötvös torsion balance. The vector equation (3) written out in Cartesian coordinates becomes,

$$\frac{I}{r}\delta H_x = H_x U_{xx} + H_y U_{yx} + H_z U_{zx}$$

$$\frac{I}{r}\delta H_y = H_x U_{xy} + H_y U_{yy} + H_z U_{zy}$$

$$\frac{I}{r}\delta H_z = H_x U_{xz} + H_y U_{yz} + H_z U_{zz}$$

where

$$U_{xx} = \frac{\partial^2 U}{\partial x^2}; \qquad U_{xy} = U_{yx} = \frac{\partial^2 U}{\partial x \partial y} \text{ etc,,}$$

so that if, for example, we wish to find $\partial^2 U/\partial x \partial y = \partial^2 U/\partial y \partial x$ we can put $H_x = H_z = O$, leaving,

$$\delta H_x = r H_y \frac{\partial^2 U}{\partial y \partial x} = r H \frac{\partial^2 U}{\partial y \partial x}.$$

Thus we require the x component of the magnetic anomaly due to a magnetizing field along the y axis, and this can be found by winding the model with a coil having the yz plane as the plane of its coils and by using a search coil with its axis parallel to the y axis. The arrangement is easy to calibrate since the mutual inductance is read directly in microhenries. The modification of the method enabling direct measurements of the vertical gravitational anomaly itself to be measured is very simple. The problem is in fact that of measuring electromagnetically a solid angle instead of its gradient, and this may be done by using a long thin solenoid, evenly wound along its length, in place of a small search coil. The solenoid, which theoretically should be infinitely long, but which in practice need extend only to a conveniently large distance from the model winding, itself automatically performs the integration of the gradient and, consequently, the mutual inductance recorded by the bridge will be a measure of the solid angle which is subtended at the near end of the solenoid. This will apply irrespectively of the direction of the axis of the long solenoid. In other words we get a measure of the magnetic potential itself instead of its gradient, the magnetic field. The use of a long solenoid for this purpose is due to a suggestion of Mr. D. L. Herbage and it has the great advantage that the isogams (lines of equal gravitational anomaly) can be traced out immediately by moving the end of the long solenoid about so that no sound is heard in the telephones for a given setting of the bridge. The fixed settings in the bridge in microhenries give the relative values to be assigned to the gravitational contours. In this way the gravitational and magnetic anomaly contours for any trial structure can be rapidly and accurately obtained.

In conclusion I wish to acknowledge my indebtedness to Dr. J. H. Jones, to whom the solid angle conception is due, for the many helpful discussions we have had on this subject, and I also wish to express my thanks to the Chairman, Lord Cadman, and the Directors of the Anglo-Iranian Oil Company for permission to publish this paper.

DIRECT METHOD OF INTERPRETATION IN RESISTIVITY PROSPECTING*

C. L. PEKERIS†

ABSTRACT

A method is given of determining the variation of conductivity with depth for the case of a horizontally layered earth.

PROCEDURE

Let the medium consist of layers having thicknesses d_1, d_2, $\cdots$ and resistivities ρ_1, ρ_2, $\cdots$. Further, let $k_i=(\rho_{i+1}-\rho_i)/(\rho_{i+1}+\rho_i)$. These constants can be determined as follows.

1. Determine the potential $\varphi(r)$ around a single electrode as a function of the distance r from the electrode. $\varphi(r)$ can be obtained directly by measuring the potential distribution around one electrode with the other electrode removed to a great distance. If the electrodes are a distance L apart and if $\psi(r)$ denote the potential measured along a line passing through the electrodes at a point which is at a distance r from the positive electrode and $(r+L)$ from the negative electrode, then

$$\varphi(r) = \sum_{n=0}^{\infty} \psi(r + nL).$$

2. Determine Slichter's "kernel" $k(\lambda)$[1] from

$$k(\lambda) = \lambda \int_0^{\infty} \varphi(r) J_0(\lambda r) r dr. \tag{1}$$

$k(\lambda)$ can be obtained either by numerical integration or by mechanical integration, say, with the aid of a planimeter.

3. Plot $ln|f_1(\lambda)|$ against λ, where

$$f_1(\lambda) = \frac{k(\lambda) + 1}{k(\lambda) - 1}. \tag{2}$$

For large values of λ the points will lie on a straight line whose slope is $2d_1$ and whose intercept with the axis of ordinates is $ln(1/k_1)$ or $ln(-1/k_1)$ depending on whether $f_1(\lambda)$ is positive or negative for large

* Manuscript received March 18, 1939.

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values of λ. If one plots $|f_1(\lambda)|$ on semilogarithmic paper with the linear scale for λ, the intercept of the asymptotic straight line with the axis of ordinates is $|1/k_1|$. In determining the slope of the straight line one should remember, however, to use the same linear scale for the ordinate as for the abscissa. If all the points lie on the straight line, then $d_2 = \infty$; otherwise proceed with the next step.

TABLE 1

THE KERNEL $k_1(\lambda)$ WAS COMPUTED FOR THE CASE $d_1 = 1.00$, $\rho_2 = 20\rho_1$, $d_2 = 5.00$, $\rho_3 = \rho_1$, $d_3 = \infty$; KERNEL $k_2(\lambda)$ WAS COMPUTED FOR THE CASE $d_1 = 1.00$, $d_2 = 2.00$, $d_3 = \rho_1$, $\rho_2 = 20\rho_1$, $\rho_3 = \infty$, KERNEL $k_3(\lambda)$ WAS COMPUTED FOR A MEDIUM WHICH HAS A CONTINUOUS VARIATION OF CONDUCTIVITY WITH DEPTH AS SHOWN BY CURVE *A* OF FIG. 5

λ	$k_1(\lambda)$	$k_2(\lambda)$	$k_3(\lambda)$	λ	$k_1(\lambda)$	$k_2(\lambda)$	$k_3(\lambda)$
0	1.00	0	0	.30	2.94	2.67	1.176
.005			.165	.35		2.45	
.010			.286	.40	2.36	2.28	1.143
.015			.469	.45		2.09	
.020	2.830		.638	.50	2.00	1.952	1.120
.03			.812	.60		1.730	1.102
.04			.967	.70	1.57	1.565	1.089
.05	4.55	1.85	1.072	.80	1.45	1.442	1.079
.07			1.190	.90	1.35	1.349	
.08	5.09			1.00	1.28	1.278	1.065
.10	5.06	2.91	1.250	1.10		1.222	
.15	4.51	3.19		1.20		1.178	
.20	3.88	3.12	1.225	1.30		1.144	
.25		2.91		1.40		1.116	
				1.50		1.094	

4. With k_1 and d_1 thus determined, compute

$$f_2(\lambda) = \frac{(1 - k_1^2)}{1 - k_1\xi_1 f_1(\lambda)}, \qquad \xi_1 = e^{-2\lambda d_1} \tag{3}$$

and plot $ln|f_2(\lambda) - 1|$ against λ. For large λ the points will again lie on a straight line whose slope is $2d_2$ and whose intercept with the axis of ordinates is $ln(k_1/k_2)$ or $ln(-k_1/k_2)$, depending again on the sign of $[f_2(\lambda) - 1]$. If all the points lie on a straight line, then $d_3 = \infty$, otherwise proceed with the next step.

5. Compute $f_3(\lambda)$ from

$$f_3(\lambda) = \frac{(1 - k_2^2)}{1 + y_2 - y_2 f_2}, \qquad y_2 = \frac{k_2}{k_1} e^{-2\lambda d_2} \tag{4}$$

and plot $ln|f_3(\lambda) - 1|$ against λ. The slope of the asymptotic straight line is $2d_3$ and its intercept with the axis of ordinates is $ln|k_2/k_3|$.

6. Continue, if necessary, with $f_4(\lambda)$, and the f's of higher order where

$$f_m(\lambda) = \frac{(1 - k_{m-1}^2)}{1 + y_{m-1} - y_{m-1} f_{m-1}} \cdot \tag{5}$$

In each step the slope of the asymptotic straight line is $2d_m$ and the intercept with the axis of ordinates is $ln\left| k_{m-1}/k_m \right|$.

Examples

We shall illustrate the procedure with some examples. In Table 1 are given three kernels $k(\lambda)$ representing conductivity functions

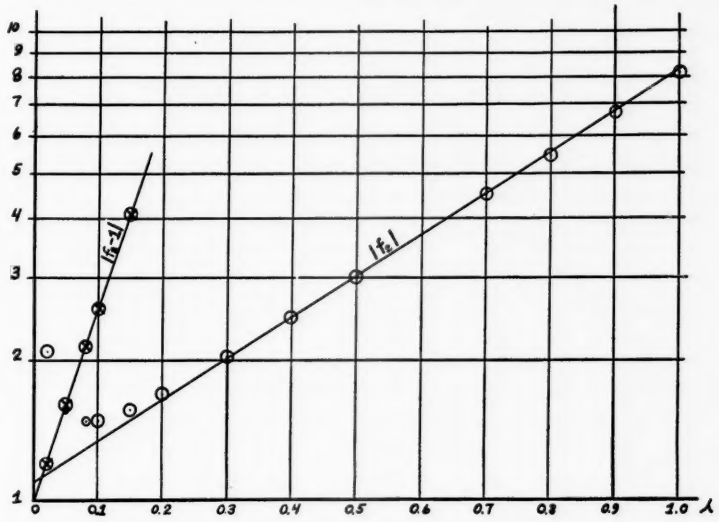

FIG. 1. Analysis of $k_1(\lambda)$.

unknown (to me), which were submitted for analysis by Prof. L. B. Slichter. The treatment of the first kernel is shown in Fig. 1, where the scale of ordinates is logarithmic. It is seen that beyond $\lambda = 0.3$, the points of $\left| f_1(\lambda) \right|$ lie very closely on a straight line. This in itself implies that right below the surface there is a layer of uniform conductivity. Also, the fact that for values of λ less than 0.3 the points deviate from the straight line indicates that the top uniform layer is limited. The intercept of the straight line with the axis of ordinates is 1.10* and, since $f_1(\lambda)$ is positive, it follows that $1/k_1 = +1.10$, $k_1 = .909$, $\rho_2/\rho_1 = (1+k_1)/(1-k_1) = 21$. Taking two significant points, say $\lambda = 0.4$ and 0.7, we have $2d_1 = (1/0.3)\,(ln\ 4.51 - ln\ 2.47) =$

* The last figures of the numbers in this section are uncertain.

2.00. Hence $d_1=1.00$, $\xi_1=e^{-2\lambda}$. With the above constants one then computes $f_2(\lambda)$ from (3). The points of $|f_2-1|$ are seen to lie on a straight line passing through the origin and since (f_2-1) is negative, it follows that $k_2=-k_1=-.909$, $\rho_3=\rho_1$. The slope of the curve is found to be 9.38 giving a value of 4.69 for d_2. Further, since all the points lie on the line, it follows that $d_3=\infty$. Actually the kernel $k_1(\lambda)$ was that computed for the case, $d_1=1.00$, $\rho_2=20\rho_1$, $d_2=5.00$, $\rho_3=\rho_1$, $d_3=\infty$.

Had we treated the kernel $k_1(\lambda)=1/k_1(\lambda)$, we would have obtained the same numerical values for $f_1(\lambda)$, but with opposite sign. Hence, for the reciprocal kernel $k_1=-.909$ and $\rho_2/\rho_1=1/21$. $f_2(\lambda)$ would have agreed both in absolute value and in sign with the corresponding function for $k_1(\lambda)$, with the result that $\rho_3=\rho_1$. This illustrates a theorem of Langer[1] that to the reciprocal of a given kernel corresponds a reciprocal resistivity distribution. Furthermore, it is clear from the preceding discussion that the accuracy obtainable in the determination of the resistivity from a given kernel by our method is the same as for the reciprocal kernel.

The example just treated was well suited for analysis by our method because the intermediate layer was five times as thick as the surface layer. For this reason we could dispense with the knowledge of $k(\lambda)$ for values of λ greater than 1.00, since it is seen from Fig. 1 that even when $\lambda \geqq 0.4$ the points do not show any systematic deviation from a straight line. Actually, with $\varphi(r)$ given, one can determine $k(\lambda)$ with increasing accuracy as λ becomes large, since for large values of λ, $k(\lambda)$ depends essentially on the variation of the potential in the vicinity of the electrode. In this region it should be possible under ideal geological conditions to obtain an accuracy of one part in a thousand for the potential and with it a corresponding accuracy for $k(\lambda)$ for large values of λ. The next two kernels were therefore tabulated to four places for large values of λ. The treatment of $k_2(\lambda)$ is shown in Fig. 2. Here the constants of the medium are $d_1=1.00$, $d_2=2.00$, $\rho_2=20\rho_1$, $d_3=\infty$, $\rho_3=\infty$. It will be noticed that up to $\lambda=1.0$ there is a systematic increase in the slope of the curve. The intercept of the asymptote to $|f_1|$ is 1.11, yielding $k_1=.901$, $\rho_2/\rho_1=19.2$. From the value of $ln|f_1|$ at $\lambda=1.1$, one obtains a value of 1.00 for d_1. With these values the computed points of $|f_2-1|$ are seen to align themselves on a straight line whose intercept with the axis of ordinates is .901 and since (f_2-1) is negative, it follows that $k_2=-1$, $\rho_3=\infty$.

[1] L. B. Slichter, Physics, vol. 4, p. 307 (1933); also: R. E. Langer, Bull. Amer. Math. Soc., p. 814 (October, 1933).

From the slope of the curve one finds that $d_2 = 2.09$. The accuracy here obtained is due to the detailed knowledge of the kernel for large values of λ. Had this kernel been given only up to three decimals and for values of λ only up to $\lambda = 1.00$, the first asymptote to $|f_1|$ would

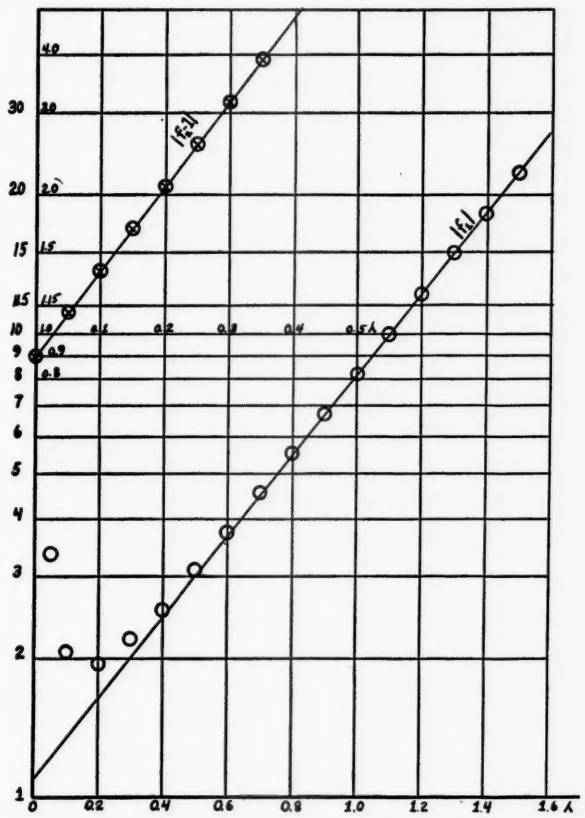

FIG. 2. Analysis of $k_2(\lambda)$. The scales inside the diagram are for $|f_2 - 1|$.

have yielded the values .974 and 12.8 for d_1 and ρ_2/ρ_1 respectively. d_2 and ρ_3 would have come out 3.16 and ∞ respectively. The kernel computed with these constants was found to agree well with $k_2(\lambda)$ for large and small values of the argument, the greatest deviation at intermediate values being only 1.7%. It is clear, therefore, that one should try to obtain the highest possible accuracy for $k(\lambda)$. The best way of judging the reliability of the results of an

analysis by our method is to recompute $k(\lambda)$ with the constants obtained and to compare it with the original kernel. For this purpose the formula for $k(\lambda)$ given by Slichter[1] on p. 319 for a medium contain-

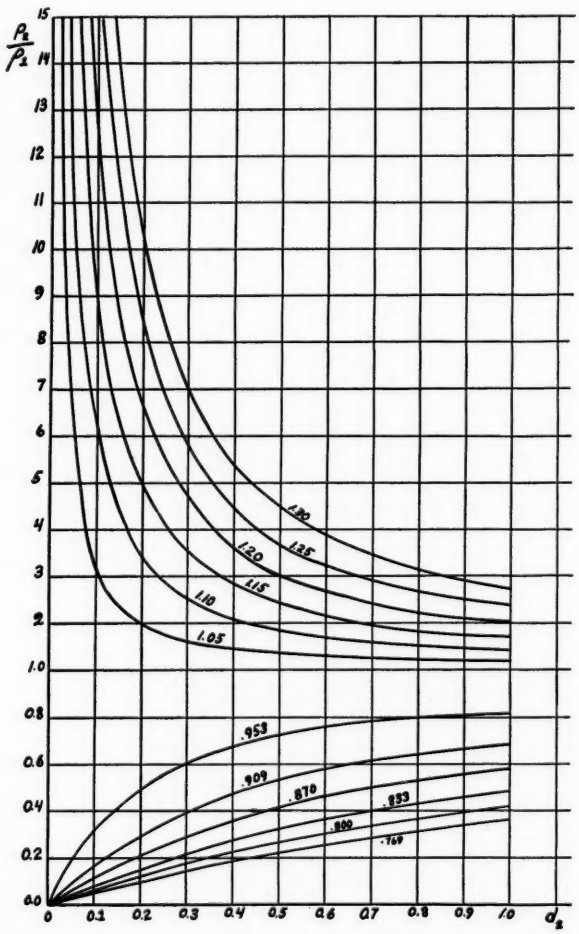

FIG. 3. The numbers on the curves are the maximum or minimum values of $k(\lambda)$ for a medium consisting of three layers of thicknesses 1, d_2, ∞ and of resistivities ρ_1, ρ_2, ρ_1. The curves in the upper left-hand corner tend to infinity as d approaches zero.

ing up to six layers should be sufficient for all cases which are likely to be met in practice. If the computed kernel agrees with the original to within the accuracy of the latter, then the information about the

structure of the medium obtained from the analysis has the maximum accuracy obtainable from the given data. If a somewhat different structure yields a kernel which also agrees with the given kernel to within its probable error, then it is impossible to distinguish between the two structures by resistivity measurements at the surface.

In this connection we shall take up the question of the detectibility of a thin subsurface layer. Let a layer of thickness d_2 and resistivity ρ_2 be bounded from above by a layer of thickness 1 and resistivity ρ_1 and from below by an infinite layer of resistivity ρ_1. The kernel starts with 1 at $\lambda=0$ and ends with 1 at $\lambda=\infty$ and at intermediate values it goes through a maximum when $\rho_2>\rho_1$ or a minimum when $\rho_2<\rho_1$. In Fig. 3 are plotted the contours of the maximum or minimum values of $k(\lambda)$ against d_2 and ρ_2/ρ_1. It is seen, for example, that with $d_2=0.2$ and $\rho_2/\rho_1=2$, the maximum value of $k(\lambda)$ is only 1.05 and similarly for $d_2=.375$ and $\rho_2/\rho_1=1.5$. An inspection of Table 2 will show that the maxima or minima of k are of the

TABLE 2

COMPARISON BETWEEN MAXIMA OR MINIMA IN THE APPARENT RESISTIVITY ρ_a AND THE KERNEL k. VALUES OF ρ_a WERE TAKEN FROM A PAPER BY W. W. WETZEL AND H. V. MCMURRY[2]

ρ_2/ρ_1	d_2	ρ_a	k
10	.333	1.62	1.50
3	.333	1.20	1.16
.333	.333	.84	.875
.100	.333	.61	.70
3	1.00	1.48	1.33
.333	1.00	.66	.74

[2] GEOPHYSICS 2, p. 329, 1937.

same order of magnitude as the maxima or minima of the apparent resistivity ρ_a. For small values of d_2, k is given by

$$k = 1 + \frac{2k_1d_2(1 - .5d_2)}{e - k_1d_2(1 - .5d_2) - ek_1^2(1 - d_2)},$$

where $k_1=(\rho_2-\rho_1)/(\rho_2+\rho_1)$. The above formula reduces to

$$k = \frac{1 + .343d_2\rho_2/\rho_1}{1 + .159d_2\rho_2/\rho_1},$$

when $\rho_2>\rho_1$ and to

$$k = \frac{1 + .159d_2\rho_1/\rho_2}{1 + .343d_2\rho_1/\rho_2}$$

when $\rho_2 < \rho_1$. If $k(\lambda)$ is known only to within 5%, then the imbedded layer could not be detected with certainty from measurements at the surface. Normal geological differences in overburden will produce fluctuations generally considerably greater than 5%. Furthermore, if the thin layer lies below *two* surface layers, the relevant function is

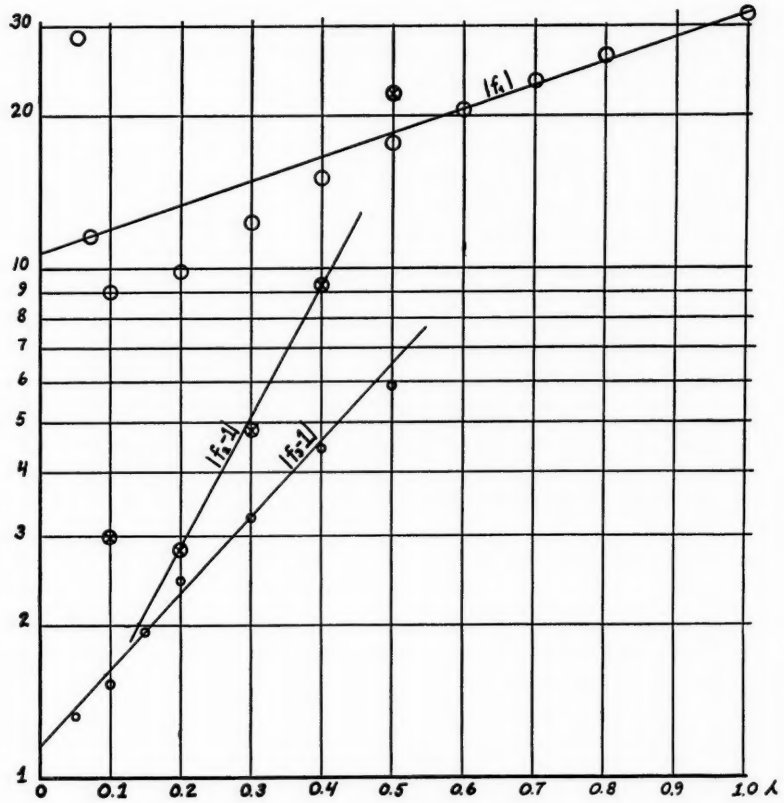

FIG. 4. Analysis of $k_3(\lambda)$.

$f_2(\lambda)$ which is known less accurately than $k(\lambda)$ on account of the uncertainty in d_1 and ρ_2 as determined by our method. For the same reason the accuracy is correspondingly reduced further for $f_3(\lambda)$ and with it the possibility of detecting a thin layer below three surface layers.

Turning now to the third kernel, we see from Fig. 4 that for large values of λ the points do not converge to a straight line. This implies

that the region right below the surface is not of uniform conductivity. In such a case the appropriate procedure is to use the Slichter-Langer[1] method of interpretation. We shall attempt, however, to approximate to the continuously varying structure by a series of layers of uniform conductivity. Passing a straight line through the points at $\lambda=0.6$ and 1.0, we get $d_1=.543$, $k_1=.0930$. The points of $|f_2-1|$ again do not lie on a straight line, but the line drawn through $\lambda=0.2$ and 0.4 passes close to the point at $\lambda=0.3$ and it yields the

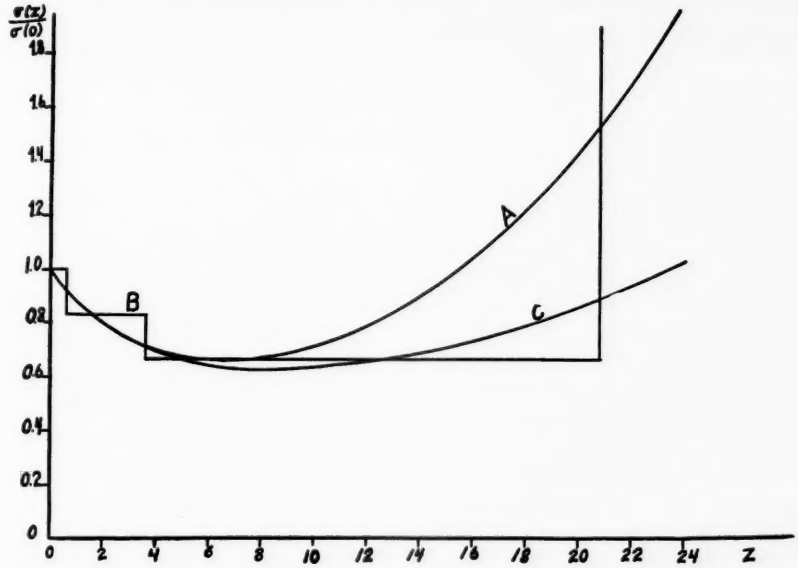

FIG. 5. Curve *A* shows actual conductivity distribution for $k_3(\lambda)$. Curve *B* was obtained from our analysis. Curve *C* represents the function $F(z)=[k_3(1/z)]^{-2}$.

values of 3.0 for d_2 and .110 for k_2. We proceed similarly with $|f_3-1|$ and represent it by a straight line which yields the values of 17.2 and $-.95$ for d_3 and k_3 respectively. Since the points for small values of λ deviate from this line, one should continue this process to obtain the structure beyond d_3. We shall, however, stop at this point and assume that $d_4=\infty$. The conductivity $\sigma(z)$ thus obtained is shown by curve *B* in Fig. 5, the real conductivity being represented by curve *A*. It will be seen that the general trend is satisfactorily reproduced.

Curve *C* represents the function $F(z)=[k(1/z)]^{-2}$. This function agrees with the power-series expansion[1] of $\sigma(z)/\sigma(0)$ up to and including z^2. In fact

$$\sigma(z)/\sigma(0) = F(z) + (z^3/3)(2a_3 - 2a_1a_2) + (z^4/3)(6a_1^2a_2 - 2a_2^2 - 8a_1a_3 + 4a_4) + \cdots, \quad (6)$$

where the a's are the coefficients in the asymptotic expansion of $k(\lambda)$ for large values of λ:

$$k(\lambda) = 1 + \frac{a_1}{\lambda} + \frac{a_2}{\lambda^2} + \cdots. \quad (7)$$

In case of a continuously varying conductivity, $F(z)$ thus faithfully reproduces the initial trend of the conductivity for small values of z.

THEORY

Let the lower boundaries of the layers occur at depths h_i and let

$$x_i = e^{-\lambda h_i}, \quad \xi_i = e^{-2\lambda h_i}, \quad r_i = \rho_i/\rho_{i+1}, \quad a_i = 1 + r_i, \quad b_i = 1 - r_i, \quad k_i = b_i/a_i.$$

It was shown by Slichter[1] that for a medium consisting of n layers superimposed on an infinite layer

$$k(\lambda) = \frac{D(\lambda) + N(\lambda)}{D(\lambda) - N(\lambda)}, \quad (8)$$

and a method of evaluating D and N for the general case was given. Here $N(\lambda)$ is the determinant

$$\begin{vmatrix} -x_1 & -x_1 & -1/x_1 & 0 & \cdots & & & & & \\ 0 & -x_1b_1 & -a_1/x_1 & 0 & \cdots & & & & & \\ 0 & x_2 & 1/x_2 & -x_2 & -1/x_2 & 0 & \cdots & & & \\ 0 & 0 & 2/x_2 & -x_2b_2 & -a_2/x_2 & 0 & \cdots & & & \\ 0 & 0 & x_3 & \cdots & & & & & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ & & & & & & & x_n & 1/x_n & -x_n \\ & & & & & & & 0 & 2/x_n & -b_nx_n \end{vmatrix} \quad (9)$$

and $D(\lambda)$ is a determinant similar to $N(\lambda)$ except for the first column which reads $1/x_1$, $2/x_1$, 0, $\cdots$ instead of $-x_1$, 0, 0, $\cdots$. We shall denote these functions by N_1 and D_1 and shall define N_m and D_m as determinants of order $(2n-2m+2)$ which have the same structure as N_1 and D_1 except that the subscripts begin with m instead of 1. Thus the minor bordered by the light lines in (9) is D_2 and the minor which is bordered to the left by the first four columns and at the top by the first four rows is D_3. We now develop N_1 and D_1 by the

minors of the second order of the first two rows and, noting that the complementary minor of

$$\begin{vmatrix} -x_1 & -1/x_1 \\ -x_1b_1 & -a_1/x_1 \end{vmatrix} \text{ is zero,}$$

while the complementary minor of

$$\begin{vmatrix} -x_1 & -1/x_1 \\ 0 & -a_1/x_1 \end{vmatrix} \text{ is } -N_2,$$

we obtain

$$N_1 = b_1\xi_1 D_2 + a_1 N_2, \tag{10}$$

$$D_1 = a_1 D_2 + (b_1/\xi_1)N_2. \tag{11}$$

It follows from (10) and (11) that

$$N_1 = k_1\xi_1 D_1 + a_1(1 - k_1^2)N_2. \tag{12}$$

Let $F_0 = D_1/(a_1a_2 \cdots a_n)$, $F_1 = N_1/(a_1a_2 \cdots a_n)$, $F_2 = N_2/(a_2a_3 \cdots a_n)$, etc. Clearly F_2 is obtained from F_1 by putting $k_1 = 0$ in the latter and similarly F_{m+1} is obtained from F_m by putting $k_m = 0$ in the latter. From the derivation of (12) it is also clear that we can advance the subscripts in it by 1. Hence:

$$N_2 = k_2\xi_2 D_2 + a_2(1 - k_2^2)N_3, \tag{13}$$

which becomes by (11)

$$N_2 = (k_2\xi_2/b_1\xi_1)N_1 - y_2 N_2 + a_2(1 - k_2^2)N_3, \tag{14}$$

where $y_2 = (k_2/k_1)e^{-2\lambda d_1}$, $d_i = h_i - h_{i-1}$, $y_m = (k_m/k_{m-1})e^{-2\lambda d_m}$. With the definition of the F's given above, equations (12) and (14) can be written as

$$F_1 = k_1\xi_1 F_0 + (1 - k_1^2)F_2, \tag{15}$$

$$(1 + y_2)F_2 = y_2 F_1 + (1 - k_2^2)F_3. \tag{16}$$

(16) is true also if we advance the subscripts in it by an integer which is less or equal to $n-3$. Thus

$$(1 + y_m)F_m = y_m F_{m-1} + (1 - k_m^2)F_{m+1}. \tag{17}$$

Since

$$F_n = k_n\xi_n, \qquad F_{n-1} = k_n\xi_n + k_{n-1}\xi_{n-1},$$

it follows that

$$(1 + y_n)F_n = F_{n-1}. \tag{18}$$

Our method is based essentially on the fact that for large values of λ

$$F_m = k_m \xi_m [1 + o(e^{-c\lambda})]. \tag{19}$$

Thus

$$f_1(\lambda) \equiv \frac{k(\lambda)+1}{k(\lambda)-1} = \frac{F_0}{F_1} \xrightarrow[\lambda\to\infty]{} \frac{1}{k_1\xi_1}[1 + o(e^{-c\lambda})],$$

and

$$ln\,|f_1(\lambda)| \to 2\lambda d_1 + ln\,|1/k_1| + o(e^{-c\lambda}). \tag{20}$$

Since the remainder in (20) decreases exponentially, $ln|f_1(\lambda)|$ should approach a straight line very rapidly. It follows now from (15) that

$$f_2(\lambda) \equiv F_1/F_2 = (1 - k_1^2)/(1 - k_1\xi_1 f_1) \xrightarrow[\lambda\to\infty]{} \frac{k_1}{k_2} e^{+2\lambda d_2}[1 + o(e^{-c\lambda})], \tag{21}$$

and similarly for $(f_2 - 1)$. Hence

$$ln\,|f_2 - 1| \to ln\,|k_1/k_2| + 2\lambda d_2. \tag{22}$$

For the f's of higher order it follows from (16) and (17) that

$$ln\,|f_m - 1| \to ln\,|k_{m-1}/k_m| + 2\lambda d_m, \tag{23}$$

with

$$f_m = (1 - k_{m-1}^2)/(1 + y_{m-1} - y_{m-1} f_{m-1}).$$

I am indebted to Prof. L. B. Slichter for helpful discussions during the preparation of the paper.

THREE-LAYER RESISTIVITY CURVES FOR THE ELTRAN ELECTRODE CONFIGURATION

S. S. WEST*

ABSTRACT

A method is described by which standard curves of apparent resistivity vs. electrode separation can be calculated for certain electrode configurations from the published data for the Wenner configuration. Data are given for plotting curves representing 15 sets of values of the three layer parameters in the case of the Eltran configuration.

The best method of interpretation for the data obtained by direct-current measurements of resistivity requires a set of standard curves of logarithm of apparent resistivity vs. logarithm of electrode separation, against which the field curves are matched. A set of standard curves for the three-layer earth, computed for the commonly used Wenner electrode configuration, was published two years ago by W. W. Wetzel and H. V. McMurry.[1] That paper states that the authors computed the general potential function from which the apparent resistivity for any electrode configuration could be obtained. However, Dr. Wetzel has informed me[2] that the elimination of errors was carried out after the Wenner apparent resistivity had been calculated from the potential function, so that to compute standard curves for any other electrode configuration would necessitate a complete reworking of the original data. For this reason it seems worthwhile to describe a method by which most of the standard curves for a great many configurations can be computed from the published data on the Wenner configuration.

Recently there has come into use a configuration in which the four electrodes are equidistant and collinear, as in the Wenner configuration, but are connected in such a manner that the first and second electrodes are the current electrodes and the third and fourth the potential electrodes (Fig. 1). Since this arrangement is now conventional for the "Eltran" method, I shall refer to it as the Eltran configuration. The formula for the new set of standard curves is derived in the following manner.

Let $\phi(r)$ be the potential on the surface of an earth of any number

* Joint Geophysical Laboratory of Stanolind Oil and Gas Company and Western Geophysical Company.

[1] W. W. Wetzel and H. V. McMurry; GEOPHYSICS, *2*, 329, Oct., 1937.

[2] Personal communication; May, 1938.

of layers at a distance r from an electrode of suitable arbitrary shape, from which a current I enters the ground. Let a be the distance between adjacent electrodes. The apparent resistivities ρ_{a1} for the Wenner configuration and ρ_{a2} for the Eltran configuration are given by the expressions:

$$\rho_{a1} = \frac{2\pi a}{I}\left[2\phi(a) - 2\phi(2a)\right] \tag{1}$$

$$\rho_{a2} = \frac{6\pi a}{I}\left[\phi(a) - 2\phi(2a) + \phi(3a)\right]. \tag{2}$$

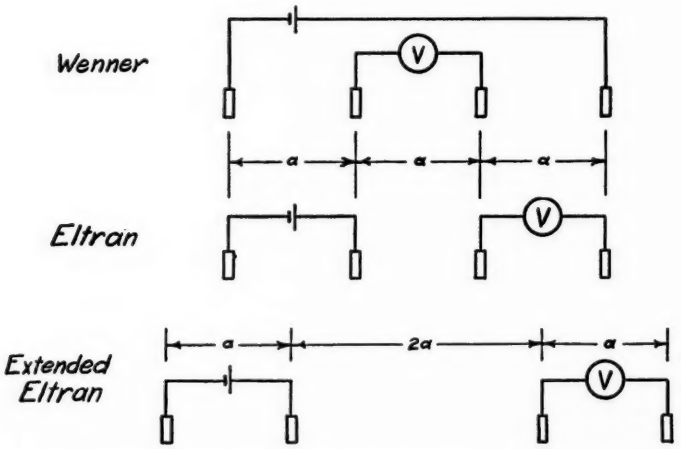

FIG. 1. The Wenner, Eltran and extended Eltran Configurations.

We now put $\rho_{a1}(a)=\alpha_1$ and $\rho_{a1}(na)=\alpha_n$. Also $(4\pi a/I)\phi(na)=\theta_n$, so that $(1/n)\alpha_n=\theta_n-\theta_{2n}$. Substituting into equation (2), we obtain

$$2/3\rho_{a2} = \theta_1 - 2\theta_2 + \theta_3. \tag{3}$$

We can further substitute $\alpha_1=\theta_1-\theta_2$, $\frac{1}{2}\alpha_2=\theta_2-\theta_4$, $\frac{1}{3}\alpha_3=\theta_3-\theta_6$, etc., whence:

$$\begin{aligned} 2/3\rho_{a2} &= \alpha_1 - \frac{1}{2}\alpha_2 + \frac{1}{3}\alpha_3 + (\theta_4 - \theta_6) \\ &= \alpha_1 - \frac{1}{2}\alpha_2 + \frac{1}{3}\alpha_3 - \frac{1}{4}\alpha_4 + \frac{1}{6}\alpha_6 - \frac{1}{8}\alpha_8 + \frac{1}{12}\alpha_{12} \\ &\quad - \frac{1}{16}\alpha_{16} + \frac{1}{24}\alpha_{24} + (\theta_{32} - \theta_{48}) \\ &= \alpha_1 - \sum_{n=1}^{\infty}\left[\frac{1}{2^n}\alpha_{(2^n)} - \frac{1}{3\cdot 2^{n-1}}\alpha_{(3\cdot 2^{n-1})}\right]. \end{aligned} \tag{4}$$

TABLE 1

LISTING VALUES OF ρ_a FOR VARIOUS SETS OF VALUES OF THE THREE-LAYER PARAMETERS

$8a/h_2$		2	4	6	8	10	15	20	25	30	40
$\rho_1/\rho_2/\rho_3$	h_1/h_2										
1/3/0.1	$\frac{3}{4}$	1.00	1.003	1.050	1.058	1.060	.845	.575	.385	.265	.141
	$\frac{1}{2}$	1.00	1.05	1.237	1.350	1.405	1.190	0.810	0.523	0.342	0.159
	$\frac{1}{4}$	1.05	1.445	1.730	1.847	1.840	1.383	0.888	0.567	0.370	0.177
1/3/1/3	$\frac{3}{4}$	1.00	1.02	1.065	1.070	1.062	.985	.816	.630	.519	.405
	$\frac{1}{2}$	1.00	1.06	1.230	1.370	1.415	1.270	.990	.735	.572	.417
	$\frac{1}{4}$	1.042	1.440	1.690	1.850	1.910	1.640	1.120	.780	.601	.432
1/3/1	$\frac{3}{4}$	1.00	1.005	1.05	1.10	1.16	1.24	1.21	1.15	1.11	1.09
	$\frac{1}{2}$	1.00	1.01	1.23	1.41	1.51	1.54	1.40	1.26	1.18	1.11
	$\frac{1}{4}$	1.04	1.42	1.70	1.90	2.01	1.95	1.73	1.51	1.37	1.19
1/10/0.1	$\frac{3}{4}$	1.00	1.01	1.083	1.240	1.360	1.460	1.372	1.135	.865	.474
	$\frac{1}{2}$	1.00	1.10	1.38	1.74	2.02	2.29	2.20	2.01	1.71	0.96
	$\frac{1}{4}$	1.06	1.69	2.50	3.06	3.40	3.72	3.39	2.65	1.98	1.10
1/10/1	$\frac{3}{4}$	1.00	1.01	1.09	1.24	1.39	1.67	1.74	1.67	1.58	1.42
	$\frac{1}{2}$	1.00	1.18	1.38	1.76	2.06	2.37	2.61	2.55	2.31	1.63
	$\frac{1}{4}$	1.08	1.71	2.44	2.97	3.44	4.02	4.06	3.40	2.73	1.70

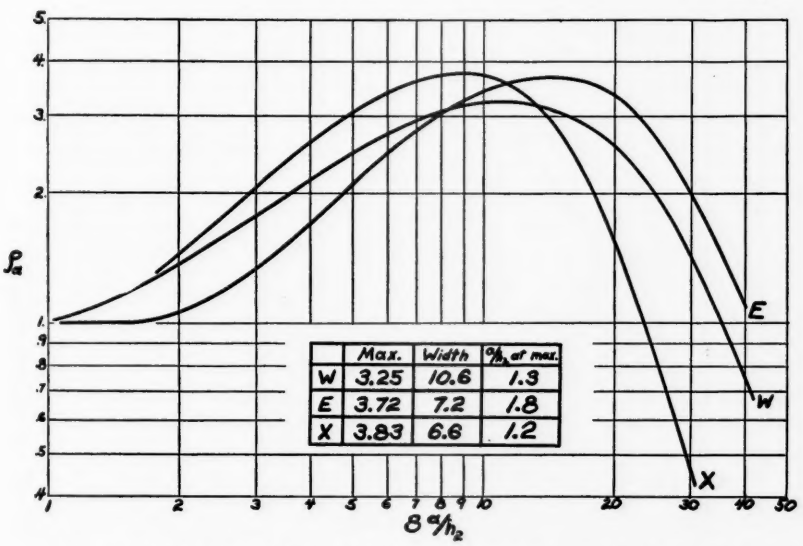

FIG. 2. A comparison of three electrode configurations for $\rho_1/\rho_2/\rho_3 = 1/10/0.1$ and $h_1/h_2 = \frac{1}{4}$. Here W denotes Wenner, E Eltran and X extended Eltran. The table gives the maximum value of ρ_a, the width of the curve (ratio of maximum a/h_2 to minimum a/h_2) for ρ_a equal to half this maximum, and the value of a/h_2 at which the maximum of ρ_a occurs.

This series converges rapidly enough when the resistivity contrasts are not too great, and even in these cases one can often extrapolate the published Wenner curves without introducing very large errors. Table 1 gives the values of ρ_{a2} for fifteen sets of values of the three-layer parameters. Time did not permit the calculation of the other cases given by Wetzel and McMurry, but it will be seen that this is not a very difficult matter.

The method can be applied to any linear configuration in which the electrode spacings are integral multiples of one quantity. The case of what we may call the "extended Eltran" configuration (Fig. 1) has been computed for $\rho_1/\rho_2/\rho_3 = 1/10/0.1$ and $h_1/h_2 = \frac{1}{4}$, where ρ_1, ρ_2, ρ_3 are the resistivities of the three layers in order of increasing depth and h_1, h_2 are the depths of the interfaces. A comparison of the Wenner, Eltran, and extended Eltran configurations for these values of the parameters is given in Fig. 2. The accompanying Table 1 shows the maxima of ρ_a, the widths (ratios of maximum a to minimum a) of the curves at a value of ρ_a equal to half the maximum value, and the values of a/h_2 (electrode spacing/depth to lower interface) at which the maxima occur. The Eltran configuration gives distinctly better resolution than the Wenner, but it is doubtful that the improvement is great enough to compensate for the longer spread and for the necessary three-fold increase in the sensitivity of the instrument used to measure difference of potential.

ADDITIONAL EVIDENCE ON THE RELATION OF TEMPERATURE TO STRUCTURE IN THE SALT CREEK OIL FIELD, NATRONA COUNTY, WYOMING*

C. E. VAN ORSTRAND†

ABSTRACT

Observations of bottom hole temperatures in approximately 100 deep wells in the Salt Creek field have been made during the past few years by the Stanolind Oil and Gas Company. These recent observations confirm and extend in a remarkable manner the results obtained in the summers of 1922 and 1923 when it was found from temperature surveys in 21 wells that the temperatures over a considerable portion of the field were definitely related to the structure.

BRIEF DESCRIPTION OF THE SALT CREEK FIELD

The Salt Creek anticline is about 30 miles long by 6 or 8 miles wide, trending in a northwest-southeast direction. The most significant feature on the anticline is the Salt Creek dome which is an oval structure about 10 miles long by 5 or 6 miles wide. The shape of the dome is outlined by the Shannon sand which completely surrounds the dome. In the north, the elevation of the sandstone above the general level is not very marked, but towards the south, the sand rises in shear cliffs, some of them 100 feet or more in height. Outside of the escarpment formed by the Shannon sand are the escarpments of the Parkman sandstone member and the stratigraphically higher sandstone which forms Little Pine Ridge. The last two escarpments do not completely surround the dome.

The rocks over the central portion of the Salt Creek dome have been removed by erosion producing anticlinal valleys.[1] Dr. W. T. Thom, Jr.,[2] estimates that erosion has progressively lowered the surface to an aggregate amount of about 2 miles. However, the present relief does not greatly exceed 100 feet so that corrections to the observed temperature readings at depths of 1000 feet or more for the effect of surface topography are not needed.

The Salt Creek structure is a closed dome with a great many transverse faults that trend in general at right angles to the major axis of the anticline (Figs. 1 and 2). The amount of downthrow on

* Approved for publication by the Director of the U. S. Geological Survey. Manuscript received April, 1939.

† U. S. Geological Survey, Washington, D. C.

[1] See bibliography.

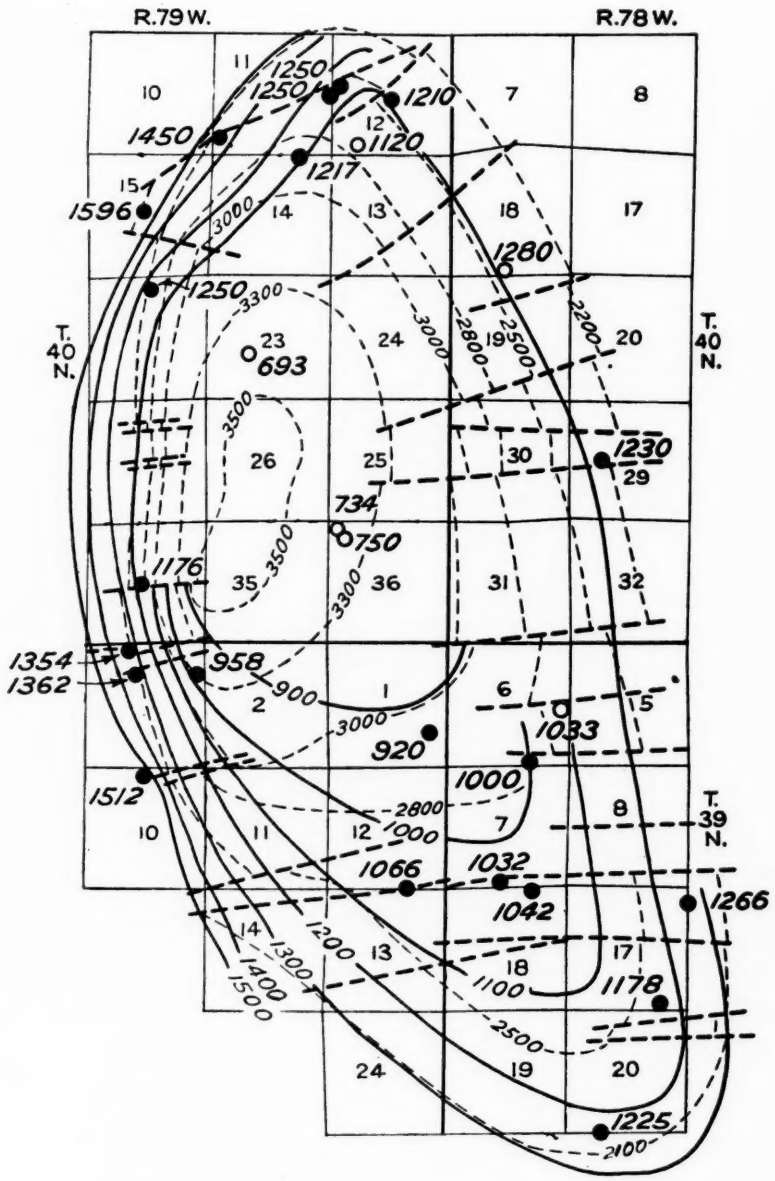

FIG. 1. Salt Creek Dome. Dashed contours show elevation of top of second Wall Creek sand above sea level. The heavy continuous lines are contours on the isogeothermal surface which represents a temperature of 80°F.

the faults ranges from almost nothing to something like 130 feet. Granite was found at a depth of 5420 feet in Stanolind Oil and Gas Company deep well, No. 29-Tp, located at the center of the SW ¼, NW ¼, Sec. 35, T. 40 N., R. 79 W., Natrona County, Wyoming.

Oil is obtained from 6 sands—the first, second, and third Wall Creek, and the Lakota, all of Upper Cretaceous age; and the Morrison and Sundance sands of Jurassic age.[3] There are also several producing horizons in the shales of the Upper Cretaceous. The relief of the oil-bearing part of the sands above edge water is roughly 500 feet in the first Wall Creek, 1400 feet in the second Wall Creek, 220 feet in the Lakota, and 150 feet in the Sundance.[3] The oil is distributed throughout the sands with remarkable uniformity. The elevation of edge water in the first Wall Creek sand falls between the exceptionally narrow limits of 3425 and 3575 feet—a range of only 150 feet around the dome.

Except for numerous faults, the physical conditions throughout the structure are uniform, and the presence of granite at moderate depths over the top of the structure is an additional factor of great importance. The field as a whole therefore offers exceptional opportunities for testing the hypothesis of a relation between temperature and structure.

THE DATA OF OBSERVATION

Not only are the structural features in the Salt Creek field highly favorable for determining the distribution of the temperatures with reference to the structure, but, the condition of the wells themselves when the first two sets of observations were made was highly satisfactory. This is shown clearly in Table 1 for the months of July and August 1922 and 1923. All of the wells were drilled with cable tools, and as shown in Table 1, practically all of them had been idle for a sufficient length of time to permit the establishment of thermal equilibrium. An exception is to be made to the observations of July and August 1929, Table 1. The wells in which these observations were made were carefully selected by Mr. Fred E. Wood, Petroleum Engineer of the Midwest Refining Company, Casper, Wyoming, for the purpose of enabling me to complete the map, Fig. 1, on which preliminary work had been done in 1922 and 1923. Unfortunately, as may be readily surmised from the history of the wells given in Table 1, repressuring and producing operations had greatly disturbed the temperatures in these wells.

TABLE 1

LOCATION AND HISTORY OF WELLS

Location			History
Sec.	*T.*	*R.*	
			July–August 1922
7	39 N	78 W	Cemented. Standing 1 year.
34	40	79	Cemented. Standing 2 years.
12	40	79	Cemented. Standing 12 days.
3	39	79	Cemented. Standing 14 months.
6	39	78	Cemented. Standing 15 months.
12	40	79	Cemented.
14	40	79	Cemented. Standing 13 days.
12	40	79	Cemented.
11	40	79	Cemented.
12	39	79	Cemented. Standing 14 days.
			July–August 1923
22	40 N	79 W	Cemented. Standing 9 months.
11	40	79	Cemented. Standing 8 months.
15	40	79	Standing several months.
3	39	79	Cemented. Standing 1 year.
3	39	79	Cemented. Standing 1 year.
1	39	79	Cemented. Standing 2 weeks.
16	39	78	
10	39	79	Standing several months.
20	39	78	Standing oil at 1330 feet.
29	40	78	Standing oil and water at 1750 feet.
18	39	78	Standing 1 year.
			July–August 1929
23	40 N	79 W	30 000 cu. ft. of gas per day.
36	40	79	10 000 cu. ft. of gas per day between casings.
36	40	79	Repressuring well about 60 ft. from location.
12	40	79	Small flow of gas probably from repressuring the field.
6	39	78	Small pumper.
18	40	78	Heavy oil in tubing. No gas.

Quite recently, Mr. H. J. Duncan, Supervisor of the Oil and Gas Leasing Division of the Conservation Branch of the U. S. Geological Survey, Casper, Wyoming, sent me an elaborate series of bottom hole pressures and temperatures that had been observed in the Salt Creek field by Mr. R. D. Hamilton, Field Superintendent of the Stanolind Oil and Gas Company at Salt Creek and compiled by Mr. E. A. Swedenborg, Gas Engineer of the Oil and Gas Leasing Division of the Geological Survey at Salt Creek, Wyoming. The results are here being published with the permission of Mr. John F. Cullen, Division Manager of the Stanolind Oil and Gas Company, Wyoming. This map, Fig. 2, supplies us with a vast amount of additional data,

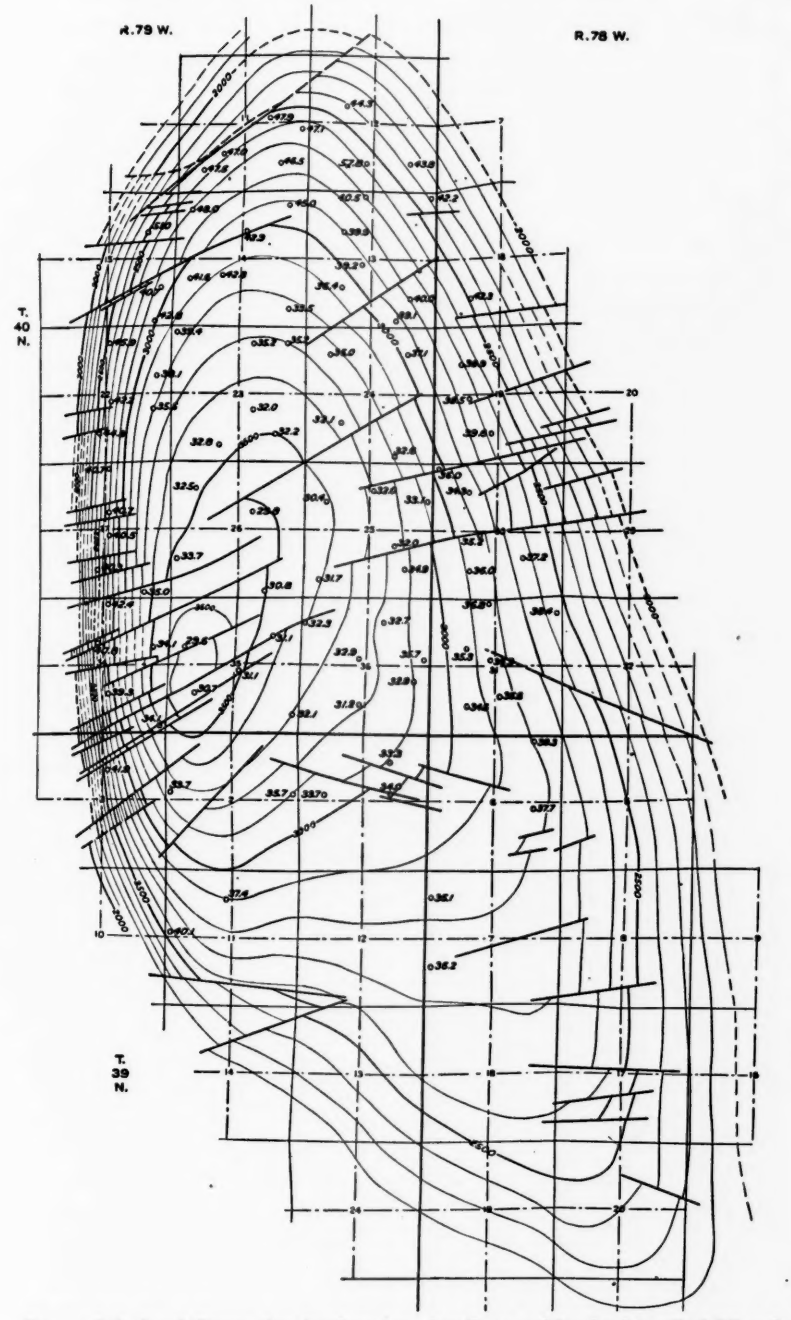

FIG. 2. Salt Creek Dome showing location of wells tested by the Stanolind Oil and Gas Company. The numbers represent feet per degree Fahrenheit. Contours are drawn on the second Wall Creek sand. The datum is sea level.

especially in the area occupied by the 6 wells in which the temperature records were vitiated by the operations of producing and repressuring the field.

STRUCTURE MAPS

Fig. 1 is a reproduction of C. H. Wegemann's map of the field which was originally drawn on a scale of 4 inches to the mile. It was prepared in 1922 by C. H. Wegemann and Harriet Van Nostrand for the Midwest Refining Company. Shortly after its completion, Mr. Wegemann who at that time was Chief Geologist of the Midwest Refining Company, kindly placed a copy of the map at my disposal for official use. It is the only map that has been published to date[4] showing the relation of temperature to structure in the Salt Creek field. My original map, which was drawn in the field, has never been published. It is based on a reproduction of Wegemann's first map[5] and is therefore less accurate than the map reproduced in Fig. 1.

Fig. 2 is based on a field map which was completed on May 1, 1926, by the Conservation Branch of the U. S. Geological Survey. The original map was drawn on the scale, 1 inch = 1000 feet.

INTERPRETATION OF THE OBSERVATIONS

The solid discs on Fig. 1 represent the location of wells tested in 1922 and 1923, and the accompanying numbers represent the depths at which a temperature of 80°F. is reached. The correlation between the 80°F. isogeothermal surface and the structure surface on the second Wall Creek sand is sufficiently obvious.

The small circles on Fig. 1 represent the locations of the 6 wells tested in 1929. At 5 of these locations, the isogeothermal surface of 80°F. is from a few feet to about 250 feet too high while at the one remaining location, Sec. 18, T. 40 N., R. 78 W., the isogeothermal surface is about 100 feet too low. These results show clearly the marked distortion in the isogeothermal surfaces which is to be expected when the wells are not in thermal equilibrium. The form of the distorted isogeothermal surface is determined largely by the distribution of wells containing the largest gas flow or other anomaly. For example, if the wells in which the abnormally high temperatures were found had been located near or beyond edge water, the resulting isogeothermal surface could not have been correlated with the structure. On the other hand, in the configuration shown in Fig. 1, the resulting isogeothermal surface obtained from the 6 wells in unstable

temperature equilibrium show a variation of temperature with structure that is too large.

In Fig. 2, slight irregularities in the sequence of values of the reciprocal gradients occur as we pass from the top to the flanks of the structure. However, taking into account the marked curvature of the depth-temperature curves in this field and the somewhat disturbed conditions under which these observations were made, the results shown in Fig. 2 prove without question that the temperatures at a given depth diminish rather rapidly in all directions from the crest to the flanks of the dome.

SUMMARY

Two different sets of observations, one set of 21 wells tested prior to production and a second set of observations from nearly 100 wells made 16 years later while the field was in a state of decline production, definitely establish the dependence of temperature on structure in this field.

Observations in 6 wells which were in unstable temperature equilibrium show that the form of the isogeothermal surface in such cases depends largely on the accidental distribution of anomalies. An important special case is that which occurs when a comparison is made between a single well near edge water in which there is a small flow of gas and a well near the top of the structure which is free from gas. Under these conditions, the observer may be led to the erroneous conclusion that the isogeothermal surfaces are planes instead of curved surfaces.

ACKNOWLEDGMENTS

The observations on which this paper is based can not be obtained without the assistance and cooperation of men in the field. I am, therefore, under great obligation to the men mentioned in this paper.

Credit is due also to Messrs. H. D. Miser and C. B. Hunt of the professional staff of the Geological Survey for the kindness of a critical review.

To all who have assisted and cooperated so generously, I extend my cordial thanks.

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DISCUSSION

Mr. Van Orstrand states in his abstract; "it was found from temperature surveys in 21 wells that the temperatures over a considerable portion of the field were definitely related to the structure"; and in commenting on Fig. 2, the author states "the results shown in Fig. 2 prove without question that the temperatures at a given depth diminish rather rapidly in all directions from the crest to the flanks of the dome." The paper mentions the fact that one of the wells in the field was carried deep and encountered granite but does not enlarge upon this fact as in the paper before the American Geophysical Union,[1] where Mr. Van Orstrand comments as follows: "The configuration of

[1] American Geophysical Union, July 1937, p. 28.

the contours in the Salt Creek Field suggests basement granite as the chief source of the heat. That this is the correct explanation was amply demonstrated by the Stanolind Oil & Gas Company's deep well No. 29-Tp which reached granite at a depth of 5420 feet. The well is located in the center of the SW/4 of the NW/4 of Sec. 35, T. 40 N., R. 79W."

If granite is the chief source of the heat as revealed by the contours in the Salt Creek Field I believe it is important to stress that fact, since there are many structures where it is presumed that there is no conformable configuration on the basement immediately under the structure in the shallower rocks. Thus the present example shows that the geothermal contours coincide with a structure conformable with a shallow basement and have their source there, whereas in the absence of the basement structure the geothermal picture may contain no reflection of the sedimentary structure.

PAUL WEAVER

AUTHOR'S REPLY

Dr. Weaver's discussion brings up the question of the causes of local variations of temperature. There are a number of angles to this problem, but for the purposes of this discussion it will perhaps suffice to consider only three of them, namely: (1) excess of radium in granite as compared with that in the over-lying sediments; (2) differences in thermal conductivity along and perpendicular to the planes of stratification; and (3) recent differential uplift and rapid erosion.

With regard to radioactivity, Jeffreys'[7] recent determination of the excess heat developed in granite over that developed in sediments is 0.35×10^{-13} calories/per gram/per second. Mathematical calculation[8] shows that this rate of generation of excess heat is not likely to produce much of a change in the temperatures over a structure. Similar calculations[8] show that the generation of heat in oil beds is likewise of no particular consequence. Haas and Hoffman[9] deduced the same conclusion from their observations in the Pechelbronn field.

The second hypothesis on differences in thermal conductivity was verified in one instance by Hull[10] who found a rate of 54.4 ft./°F. in the horizontal strata at Rose Bridge colliery and 77 ft./°F. in the inclined strata at Dukenfield. He summarizes his results as follows:

"The general inference which may be drawn from the cases just described, as far as they bear upon the temperature of coal mines is this: that in those districts where the strata are highly inclined (at angles varying from 30 to 60 degrees), the underground temperature will be lower than in the case of those where the strata are in a position approaching the horizontal."

It follows from the preceding statement that the temperature gradients in the horizontal strata over the top of a dome or an anticline should exceed those on the flanks and the same is true of an angular unconformity in which a truncated anticline is over-lain with horizontal strata.

Assuming isotropic conductivity, a special case arises when sediments of poor conductivity are arched over sediments or crystalline rocks of good conductivity. In this case, Strong[11] has shown that the highest temperatures are again found over the crest of the structure; but, when good conductors, like salt, marl and anhydrite are superposed over a poor conductor like limestone, the conditions are reversed and the isogeotherms are depressed over the highest point on the limestone fold.

The third hypothesis, "recent differential uplift and rapid erosion" was proposed

by Dr. W. T. Thom, Jr.[12] Reasoning by analogy from the flow of heat from dikes of considerable width, it appears that disturbed isogeotherms return very slowly to their normal positions. Hence, once the isogeotherms are disturbed, remnants of the abnormalities are retained throughout long intervals of time thereafter.

Dr. Washburne[13] suggested the movement of water over the dome as a possible source of excess heat. Dr. Thom[14] did not accept this explanation as applicable to Salt Creek. This point of view seems to be correct for interpolation and extrapolation[15] of the depth-temperature curves to the First Wall Creek sand shows that the highest temperatures in the sand are found over the top of the structure. Obviously, a distribution of temperatures of this kind could not exist under the conditions imposed by moving water except in the exceptional case of a very slow movement. A question arises, however, in regard to Thermopolis. Here enormous quantities of water at a temperature of 135° F. are discharged from the Big Horn spring about 6 miles west of the Warm Springs oil field in which the reciprocal gradients are about 20 feet per °F. Probably the most significant feature in this field, apart from the possibility of intrusives, is the deep syncline south of the field described by Darton.[16]

To my mind, the most important factors at Salt Creek are differences in thermal conductivity parallel to and transverse to the laminations of the strata, and recent uplift and erosion which has brought the hot rocks close to the surface of the ground. A plug provides the simplest explanation, but the evidence on this point seems to be inconclusive.

C. E. VAN ORSTRAND

GEOTHERMAL METHODS OF ESTIMATING THE AGE OF THE EARTH.*

C. E. VAN ORSTRAND†

ABSTRACT

Tables are given by means of which the temperatures in a homogeneous radioactive and non-radioactive earth, subject to a constant temperature at the surface, can be calculated rigorously with a minimum amount of labor.

The rigorous calculations lead to some unexpected results. Among them is the fact that a temperature of 76000°C. at the center of the earth is consistent with observed temperature gradients and that melting of the rocks at a depth of 500 kilometers is a possible result of radioactive heat.

INTRODUCTION

With the discovery of radioactivity, it has been generally assumed that the age of the earth could not be determined by geothermal methods. However, the assumed facts of radioactivity are not altogether consistent, and recent geothermal surveys show clearly that the simple interpretations previously given to geothermal data are probably incorrect. Hence, it seems desirable to reconsider the problem in the light of recent observations of deep earth temperatures. No pretense is made to finality. The real object of the paper is to summarize existing possibilities, rather than to reach final conclusions.

The paper consists of two parts—A.—Temperatures in a non-radioactive earth; and B.—Temperatures in a radioactive earth. The temperature for a given time and depth is the sum of the values computed from A and B.

A. TEMPERATURES IN A NON-RADIOACTIVE EARTH

The equation which represents the temperatures in a homogeneous non-radioactive sphere has been known for a number of years, but, it seems that no one has undertaken the rather difficult task of computing the coefficients which are needed in evaluating the function. In the present paper, the coefficients have been computed for the first 40 terms. This number of terms is amply sufficient for an age of the earth which equals or exceeds 1 billion years. For ages less than 1 billion years, a larger number of terms is required.

* Approved for publication by the Director of the U. S. Geological Survey. Manuscript received April, 1939.

† U. S. Geological Survey, Washington, D.C.

Formulas and table for calculation of temperatures at any depth (R–r) from the surface of the earth.

The general equation,[1]

$$v = \frac{2}{Rr} \sum_{m=1}^{m=\infty} \sin \frac{m\pi r}{R} e^{-(\kappa m^2 \pi^2 t)/R^2} \int_0^R \lambda f(\lambda) \sin \frac{m\pi\lambda}{R} d\lambda. \quad (1)$$

is the rigorous solution for the temperature (v) in a homogeneous non-radioactive sphere of radius R in which $f(\lambda)$ is the initial temperature distribution from which the time t is reckoned. Distance from the center of the earth is represented by r, and κ represents the coefficient of diffusivity of the rocks.

Let it be assumed that

$$f(\lambda) = f(r) = a + br + cr^2 + dr^3 + er^4 + fr^5 + \cdots . \quad (2)$$

Substitute in (1) and integrate term by term. Then, after substitution of the limits, R and O, we have

$$\begin{aligned} v_1 = \frac{2}{R} \cdot \frac{1}{r} \sum_{m=1}^{m=\infty} \sin \frac{m\pi r}{R} \Bigg[& a \frac{R^2}{m^2\pi^2} \{ - (m\pi \cos m\pi) \} \\ & + b \frac{R^3}{m^3\pi^3} \{ - (m^2\pi^2 - 2) \cos m\pi - 2 \} \\ & + c \frac{R^4}{m^4\pi^4} \{ - (m^3\pi^3 - 6m\pi) \cos m\pi \} \\ & + d \frac{R^5}{m^5\pi^5} \{ - (m^4\pi^4 - 12m^2\pi^2 + 24) \cos m\pi + 24 \} \\ & + e \frac{R^6}{m^6\pi^6} \{ - (m^5\pi^5 - 20m^3\pi^3 + 120m\pi) \cos m\pi \} \\ & + f \frac{R^7}{m^7\pi^7} \{ - (m^6\pi^6 - 30m^4\pi^4 + 360m^2\pi^2 - 720) \cos m\pi - 720 \} \\ & + \cdots \Bigg] e^{-(\kappa m^2 \pi^2 t)/R^2}. \end{aligned} \quad (3)$$

Multiply the terms in the large bracket by $2/R$. Since these product terms depend only on m, π, and R, they can be tabluated for a given value of R. In Table 1, the coefficients of a, b, c, and d are based

[1] (Numbers refer to Bibliography at end of article.)

TABLE 1

COEFFICIENTS OF a, b, c AND $d \div \left(\frac{1}{r} \sin \frac{m\pi r}{R} e^{-(km^2\pi^2 t)/R^2}\right)$ FOR ANY VALUE OF $R-r$

m	$[a]$	$[b]$	$[c]$	$[d \times 10^{+3}]$
1	+4056.045 456	+15 368 609.998	+64 552 789 917.	+290 478 599 315.
2	−2028.022 728	−12 920 981.597	−69 810 923 616.	−365 067 247 374.
3	+1352.015 151	+ 8 226 085.761	+51 174 508 170.	+304 552 463 505.
4	−1014.011 364	− 6 460 490.797	−39 597 277 892.	−242 318 820 744.
5	+ 811.209 091	+ 5 084 605.812	+32 128 236 658.	+199 759 845 823.
6	− 676.007 576	− 4 306 993.865	−26 977 421 811.	−168 926 769 052.
7	+ 579.435 065	+ 3 661 174.469	+23 228 881 559.	+146 167 891 133.
8	− 507.005 682	− 3 230 245.398	−20 385 115 955.	−128 632 560 071.
9	+ 450.671 717	+ 2 856 962.504	+18 156 573 523.	+114 813 529 868.
10	− 405.604 545	− 2 584 196.319	−16 364 394 557.	−103 623 470 393.
11	+ 368.731 405	+ 2 341 400.596	+14 892 514 463.	+ 94 407 594 902.
12	− 338.003 788	− 2 153 496.932	−13 662 481 872.	− 86 677 651 082.
13	+ 312.003 497	+ 1 983 076.207	+12 619 431 197.	+ 80 112 317 559.
14	− 289.717 532	− 1 845 854.514	−11 723 870 892.	− 74 462 974 776.
15	+ 270.403 030	+ 1 719 694.330	+10 946 667 509.	+ 69 555 371 156.
16	− 253.502 841	− 1 615 122.699	−10 265 867 603.	− 65 250 423 720.
17	+ 238.590 909	+ 1 517 983.718	+ 9 664 619 234.	+ 61 445 991 692.
18	− 225.335 859	− 1 435 664.622	− 9 129 774 454.	− 58 058 466 971.
19	+ 213.476 077	+ 1 358 576.375	+ 8 650 926 474.	+ 55 024 201 742.
20	− 202.802 273	− 1 292 098.160	− 8 219 731 807.	− 52 290 016 770.
21	+ 193.145 022	+ 1 229 438.766	+ 7 829 423 816.	+ 49 814 258 753.
22	− 184.365 702	− 1 174 634.691	− 7 474 457 479.	− 47 561 532 682.
23	+ 176.349 802	+ 1 122 702.817	+ 7 150 245 929.	+ 45 503 466 486.
24	− 169.001 894	− 1 076 748.466	− 6 852 962 307.	− 43 615 608 863.
25	+ 162.241 818	+ 1 033 008.233	+ 6 579 388 763.	+ 41 877 981 376.
26	− 156.001 748	− 993 921.661	− 6 326 800 043.	− 40 273 160 269.
27	+ 150.223 906	+ 956 577.646	+ 6 092 872 808.	+ 38 786 678 069.
28	− 144.858 766	− 922 927.257	− 5 875 614 211.	− 37 405 788 257.
29	+ 139.863 636	+ 890 672.749	+ 5 673 305 198.	+ 36 119 760 738.
30	− 135.201 515	− 861 398.773	− 5 484 455 096.	− 34 919 058 286.
31	+ 130.840 176	+ 833 260.155	+ 5 307 764 912.	+ 33 795 556 827.
32	− 126.751 420	− 807 561.350	− 5 142 097 506.	− 32 741 979 826.
33	+ 122.910 468	+ 782 798.357	+ 4 986 453 164.	+ 31 752 064 627.
34	− 119.295 454	− 760 057.741	− 4 839 949 455.	− 30 820 164 029.
35	+ 115.887 013	+ 738 097.529	+ 4 701 804 529.	+ 29 941 373 327.
36	− 112.667 929	− 717 832.311	− 4 571 323 189.	− 29 111 243 358.
37	+ 109.622 850	+ 698 224.671	+ 4 447 885 218.	+ 28 325 878 742.
38	− 106.738 038	− 680 051.663	− 4 330 935 539.	− 27 581 727 003.
39	+ 104.001 165	+ 662 437.881	+ 4 219 975 887.	+ 26 875 655 487.
40	− 101.401 136	− 646 049.080	− 4 114 557 720.	− 26 204 793 583.

on $R = 6371.22130$ kilometers which is the radius of a sphere having the same volume as the earth. To complete the calculation of the co-

efficients of a, b, c, d, in Table 1, it is necessary to multiply each tabular value by the corresponding value of

$$\frac{1}{r}\sin\frac{m\pi r}{R}\,e^{-(\kappa m^2\pi^2 t)/R^2} \text{ for } m = 1, 2, 3, \cdots.$$

TRANSFORMATION OF UNITS

Let κ be expressed in kilometers and millions of tropical years. To change to the C.G.S. system, we have,[2]

$$[\kappa] = \frac{[(\text{Length})^2]}{[\text{Time}]} = \frac{(100\,000)^2}{31556927.(1\,000\,000)} = \frac{1}{3155.6927},$$

or conversely, to change from C.G.S. units to kilometers and millions of tropical years, we multiply by 3155.6927.

Assuming $\kappa = 0.0064$ C.G.S. units, a value sometimes used for crustal rocks, we have for the new unit expressed in kilometers and millions of years, $\kappa = 20.196434$, and $(\kappa\pi^2 t)/R^2 = 0.000\,004\,910\,538t$.

CALCULATION OF TEMPERATURES AT SPECIFIED DEPTHS

In Table 2, the term in $1/r \sin (m\pi r)/R$ has been included for depths of 1, 50 and 100 kilometers. The complete table contains coefficients for many additional depths. (See Table 5.)

To use the table, multiply the tabular values in each column by the assumed value of the constant at the head of the table, and then tabulate the sum (Σ) of the products on each horizontal line in a vertical column as shown in the second column of Table 3. Multiply each value in this column by the corresponding value of the exponential factor, $e^{-(\kappa m^2\pi^2 t)/R^2}$. These products are tabulated in the last three columns of Table 3. The algebraic sum of the numbers in each column is the computed temperature (v_1)at that particular time (t) and depth ($R-r$).

Six quantities are involved in the application of Table 2—a, b, c, d, κ, t. The values assumed for κ must depend largely on judgment. The values of a, b, c, d are known when the law of the initial distribution of temperatures from the surface to the center of the solidified earth is known. With these five constants (a, b, c, d, κ) at our disposal, it is possible to compute the temperatures for given values of r and t. The reciprocal of the computed temperature for a depth of 1 kilometer ($R-r=1$) is assumed to be the reciprocal gradient at the surface of the earth, and comparison of this value of the reciprocal gradient for a given value of t with our assumed reciprocal gradient in a uniformly cooling globe enables us in a few trials to evaluate t.

TABLE 2

COEFFICIENTS OF a, b, c AND $d \div [e^{-(\kappa m^2\pi^2 t)/R^2}]$, $R-r=1$ Kilometer

m	$[a]$	$[b]$	$[c]$	$[d]$
1	0.00031 39614	1.18961 94	4996.7596	224 84725.
2	.00031 39611	2.00031 54	10807.528	565 16579.
3	.00031 39612	1.91023 87	11883.601	707 22323.
4	.00031 39611	2.00031 54	12260.222	750 27441.
5	0.00031 39609	1.96788 64	12434.537	773 12716.
6	.00031 39606	2.00031 28	12529.222	784 55272.
7	.00031 39605	1.98376 69	12586.313	791 99456.
8	.00031 39603	2.00031 04	12623.363	796 54955.
9	.00031 39600	1.99030 04	12648.761	799 84743.
10	0.00031 39599	2.00030 78	12666.927	802 10173.
11	.00031 39595	1.99360 58	12680.360	803 84163.
12	.00031 39593	2.00030 40	12690.576	805 11678.
13	.00031 39590	1.99550 49	12698.522	806 14413.
14	.00031 39586	2.00029 98	12704.825	806 93400.
15	0.00031 39582	1.99669 44	12709.904	807 59017.
16	.00031 39578	2.00029 50	12714.058	808 11256.
17	.00031 39574	1.99748 71	12717.496	808 55659.
18	.00031 39570	2.00028 96	12720.375	808 91975.
19	.00031 39565	1.99804 07	12722.806	809 23381.
20	0.00031 39561	2.00028 33	12724.878	809 49612.
21	.00031 39556	1.99844 21	12726.661	809 72650.
22	.00031 39551	2.00027 78	12728.206	809 92229.
23	.00031 39544	1.99874 03	12729.535	810 09514.
24	.00031 39538	2.00026 90	12730.706	810 24449.
25	0.00031 39532	1.99896 78	12731.735	810 37824.
26	.00031 39525	2.00026 09	12732.644	810 49474.
27	.00031 39518	1.99914 45	12733.449	810 59989.
28	.00031 39512	2.00025 24	12734.168	810 69244.
29	.00031 39504	1.99928 36	12734.808	810 77643.
30	0.00031 39497	2.00024 29	12735.382	810 85091.
31	.00031 39489	1.99939 42	12735.895	810 91887.
32	.00031 39481	2.00023 27	12736.359	810 97961.
33	.00031 39473	1.99948 28	12736.777	811 03530.
34	.00031 39464	2.00022 19	12737.155	811 08534.
35	0.00031 39455	1.99955 46	12737.497	811 13143.
36	.00031 39446	2.00021 07	12737.806	811 17298.
37	.00031 39437	1.99961 25	12738.088	811 21142.
38	.00031 39427	2.00019 85	12738.343	811 24619.
39	.00031 39418	1.99965 95	12738.575	811 27845.
40	0.00031 39407	2.00018 60	12738.786	811 30777.

TABLE 2—*Continued*

$R-r=50$ kilometers

m	[a]	[b]	[c]	[d]
1	0.01581 8126	59.93587 1	2 51748.710	11328 34229.
2	.01581 3313	100.75012 1	5 44344.014	28465 77031.
3	.01580 5308	96.16447 1	5 98239.499	35602 74813.
4	.01579 4098	100.62769 6	6 16761.611	37743 23720.
5	0.01577 9691	98.90607 7	6 24960.513	38857 41265.
6	.01576 2094	100.42378 6	6 29017.579	39387 71753.
7	.01574 1314	99.46187 6	6 31051.088	39708 93173.
8	.01571 7357	100.13875 7	6 31945.841	39876 55142.
9	.01569 0231	99.46575 1	6 32124.929	39972 57207.
10	0.01565 9946	99.77298 9	6 31811.356	40007 88681.
11	.01562 6516	99.22652 4	6 31131.820	40009 11899.
12	.01558 9952	99.32703 3	6 30162.854	39978 85407.
13	.01555 0264	98.83658 8	6 28952.904	39928 00700.
14	.01550 7470	98.80153 3	6 27533.984	39857 18247.
15	0.01546 1585	98.33174 5	6 25928.052	39771 60894.
16	.01541 2627	98.19725 2	6 24150.724	39671 36629.
17	.01536 0608	97.72859 6	6 22213.332	39559 25536.
18	.01530 5554	97.51507 0	6 20124.381	39435 22387.
19	.01524 7479	97.03601 3	6 17890.485	39300 91324.
20	0.01518 6412	96.75598 3	6 15516.887	39156 25098.
21	.01512 2367	96.25939 6	6 13007.816	39002 27077.
22	.01505 5368	95.92109 0	6 10366.868	38838 91828.
23	.01498 5447	95.40244 5	6 07597.047	38666 88379.
24	.01491 2620	95.01160 6	6 04701.071	38486 13744.
25	0.01483 6917	94.46799 7	6 01681.249	38297 16832.
26	.01475 8369	94.02884 0	5 98539.776	38099 97022.
27	.01467 6999	93.45843 4	5 95278.752	37894 90778.
28	.01459 2840	92.97422 3	5 91900.020	37681 99557.
29	.01450 5925	92.37590 2	5 88405.453	37461 52087.
30	0.01441 6274	91.84928 0	5 84796.809	37233 51475.
31	.01432 3932	91.22245 2	5 81075.817	36998 21129.
32	.01422 8926	90.65563 4	5 77244.195	36755 65795.
33	.01413 1289	90.00007 2	5 73303.637	36506 05631.
34	.01403 1065	89.39500 7	5 69255.831	36249 46536.
35	0.01392 8277	88.71077 5	5 65102.453	35986 06304.
36	.01382 2971	88.06920 8	5 60845.196	35715 91941.
37	.01371 5185	87.35659 2	5 56485.777	35439 19846.
38	.01360 4952	86.68014 7	5 52025.903	35155 97873.
39	.01349 2312	85.93961 1	5 47467.309	34866 41416.
40	0.01337 7310	85.22982 4	5 42811.726	34570 59102.

TABLE 2—*Continued*

$R-r=100$ kilometers

m	[a]	[b]	[c]	[d]
1	0.03187 8791	120.79073	5 07357.462	22830 38145.
2	.03184 0046	202.85998	10 96034.542	57315 71827.
3	.03177 5526	193.33231	12 02720.981	71576 97233.
4	.03168 5339	201.87431	12 37316.708	75718 62082.
5	0.03156 9606	197.87624	12 50325.989	77740 00429.
6	.03142 8500	200.23794	12 54216.615	78536 32618.
7	.03126 2224	197.53112	12 53266.389	78861 87075.
8	.03107 1020	197.96035	12 49268.780	78830 37895.
9	.03085 5169	195.60149	12 43086.977	78606 90447.
10	0.03061 4981	195.05483	12 35182.532	78214 87092.
11	.03035 0808	192.72404	12 25824.199	77708 24394.
12	.03006 3036	191.53826	12 15180.759	77093 61529.
13	.02975 2073	189.10247	12 03365.555	76393 61991.
14	.02941 8376	187.43101	11 90460.491	75610 88952.
15	0.02906 2430	184.82965	11 76528.138	74756 86121.
16	.02868 4742	182.75684	11 61619.211	73833 16116.
17	.02828 5865	179.96276	11 45777.487	72846 56733.
18	.02786 6358	177.54275	11 29041.760	71798 52515.
19	.02742 6842	174.54631	11 11448.206	70693 64206.
20	0.02696 7938	171.81871	10 93031.255	69533 44006.
21	.02649 0305	168.62049	10 73824.284	68321 45522.
22	.02599 4622	165.61750	10 53860.375	67059 33619.
23	.02548 1605	162.22455	10 33172.286	65750 07559.
24	.02495 1971	158.97453	10 11792.953	64395 45902.
25	0.02440 6483	155.39826	9 89755.455	62998 19595.
26	.02384 5908	151.92754	9 67093.454	61560 20368.
27	.02327 1032	148.18247	9 43840.703	60084 04706.
28	.02268 2670	144.51632	9 20031.528	58571 75649.
29	.02208 1655	140.61929	8 95700.610	57025 82648.
30	0.02146 8809	136.78254	8 70883.145	55448 38803.
31	.02084 5003	132.75211	8 45614.616	53841 90377.
32	.02021 1091	128.76934	8 19930.892	52208 57971.
33	.01956 7961	124.62540	7 93868.210	50550 87008.
34	.01891 6486	120.52114	7 67463.039	48871 04109.
35	0.01825 7578	116.28457	7 40752.136	47171 54054.
36	.01759 2123	112.08329	7 13772.462	45454 68142.
37	.01692 1022	107.77568	6 86561.044	43722 90185.
38	.01624 5205	103.50177	6 59155.037	41978 53825.
39	.01556 5558	99.14518	6 31591.681	40224 02251.
40	0.01488 3001	94.82289	6 03908.101	38461 69691.

TABLE 3

EXAMPLE OF FINAL SUMMATIONS. $R-r=1$ Kilometer. $\kappa=0.0064$ C.G.S. Units

Millions of years (t)		1000	2000	3000
m	Σ	$\Sigma\times$Exp.	$\Sigma\times$Exp.	$\Sigma\times$Exp.
1	+10.73553	+10.68294	+10.63061	+10.57854
2	− 1.55490	− 1.52466	− 1.49500	− 1.46592
3	− 0.34264	− 0.32783	− 0.31365	− 0.30009
4	− 0.07598	− 0.07024	− 0.06493	− 0.06003
5	+ 0.08863	+ 0.07839	+ 0.06933	+ 0.06132
6	.19790	.16583	.13896	.11645
7	.24175	.19005	.14941	.11745
8	.29376	.21454	.15668	.11443
9	.30906	.20764	.13949	.09372
10	+ 0.33813	+ 0.20693	+ 0.12664	+ 0.07750
11	.34409	.18994	.10485	.05788
12	.36222	.17860	.08806	.04342
13	.36454	.15898	.06933	.03023
14	.37676	.14390	.05496	.02099
15	+ 0.37746	+ 0.12504	+ 0.04142	+ 0.01372
16	.38618	.10986	.03125	.00889
17	.38615	.09342	.02260	.00547
18	.39265	.07999	.01630	.00332
19	.39226	.06663	.01132	.00192
20	+ 0.39729	+ 0.05573	+ 0.00782	+ 0.00110
21	.39673	.04550	.00522	.00060
22	.40069	.03721	.00345	.00032
23	.40008	.02978	.00222	.00016
24	.40330	.02384	.00141	.00008
25	+ 0.40267	+ 0.01871	+ 0.00087	+ 0.00004
26	.40533	.01466	.00053	.00002
27	.40470	.01128	.00031	.00001
28	.40693	.00866	.00018	.00000
29	.40635	.00654	.00010	.00000
30	+ 0.40822	+ 0.00492	+ 0.00006	+ 0.00000
31	.40767	.00364	.00003	.00000
32	.40929	.00268	.00002	.00000
33	.40876	.00195	.00001	.00000
34	.41016	.00140	.00000	.00000
35	+ 0.40966	+ 0.00100	+ 0.00000	+ 0.00000
36	.41089	.00071	.00000	.00000
37	.41043	.00049	.00000	.00000
38	.41151	.00034	.00000	.00000
39	.41108	.00023	.00000	.00000
40	+ 0.41204	+ 0.00016	+ 0.00000	+ 0.00000
Temperature (v_1)		+11.23938	+ 9.99986	+ 9.52154
Meters per °C.		88.97	100.00	105.03
Feet per °F.		162.17	182.27	191.43
Millions of years (t)		1000	2000	3000

ACCURACY OF TABLES 1 AND 2

On account of the fundamental importance of Tables 1 and 2, an extra effort was made to obtain absolute accuracy in the calculations. All of my calculations were carefully duplicated by H. C. Spicer, Junior Geophysicist of the Geological Survey. Further checks were made by the method of differences. No irregularities of any consequence were discovered. The calculations were made with 9 and 10 digit computing machines. Eight place values of the natural sine[3]

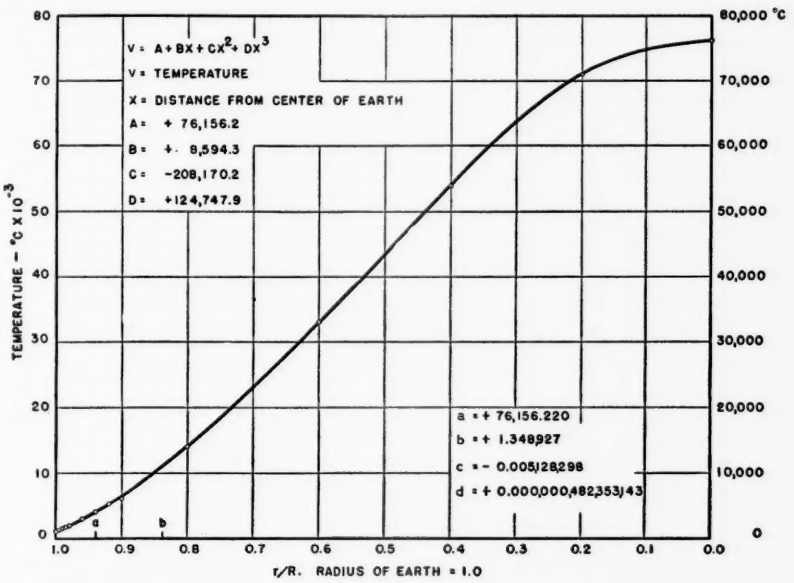

FIG. 1. Adjusted melting point curve of the King-Barus data.

were used in calculating Table 2. For convenience in tabulation, some extra digits which are not always reliable, have been included in both of the tables.

DATA OF OBSERVATION AND CALCULATION

Attempts to evaluate the constants (a,b,c,d,) or their equivalents, have been made by King and Barus, Adams, Jeffreys, Lowan, Gutenberg, and others.[4] For purposes peculiar to the requirements of this paper, the melting point curve of King and Barus, Fig. 1, has been used; for, in the first place, the high temperatures assumed by them are especially suitable for testing the tables for any case that is likely

to arise; and, secondly, the validity of extending the Barus melting point curve to the center of the earth on the basis of the Laplacian law of the distribution of pressures, or densities, is a procedure that deserves careful consideration; notwithstanding the fact that zones of discontinuity in density probably exist at various depths from the surface of the earth.

The curve shown in Fig. 1 is based on a least square adjustment of the Barus-King data,[4] tabulated in Table 4. The constants (A,B,C,D) are used when distances (r) from the center of the earth are expressed as fractions of the earth's radius while in the second set of constants (a,b,c,d) distances (r) from the center of the earth are expressed in kilometers.

TABLE 4

KING-BARUS MELTING POINT DATA

r Earth rad.	v_m °C.	v_c °C.	$v_m - v_c$ °C.	p Atm.	v_c °C.	$v_m - v_c$ °C.	$\frac{v_c - 1225}{p}$
1.000	1170	1328	−158	0	1225	− 55	.....
0.995	1380	1500	−120	8600	1440	− 60	0.025000
.990	1600	1680	− 80	17400	1660	− 60	.025000
.985	1830	1868	− 38	26400	1885	− 55	.025000
.980	2060	2064	− 4	35600	2115	− 55	.025000
.960	3030	2926	+104	74500	3088	− 58	.025007
.940	4080	3909	+171	116400	4136	− 56	.025009
.920	5210	5007	+203	162000	5277	− 67	.025012
.900	6100	6214	−114	199000	6202	−102	.025010
.800	14000	13674	+326	497000	13657	+343	.025014
.600	33000	33317	−317	1260000	32742	+258	.025013
.400	54000	54271	−271	2100000	53754	+246	.025014
.200	71000	70546	+454	2770000	70513	+487	.025050
.000	76000	76156	−156	3020000	76766	−766	.025014

The probable error of an observation of weight unity is ±172°C.

A least square adjustment of the pressure data gives

$$v_m = 1224.65 + 0.025014p \cdots \quad (4)$$

for the relation between pressure (p) and melting point temperature (v_m). The average coefficient, 0.025014, agrees approximately with the values used by Barus[5] in evaluating the temperatures as a function of the pressures. The probable error resulting from equation (4) is ±205°C. The systematic deviations in the last two columns of Table 4 show that Barus varied the coefficient of p slightly with increasing depth.

RESULTS OF SOME CALCULATIONS

Table 3 is obtained from Table 2 in the manner previously described; in this case, by substitution of the values of (a,b,c,d) computed from the King-Barus data (Fig. 1). The summations at the bottom of the last three columns are the temperatures in degrees Centigrade at a depth of 1 kilometer for the respective time intervals of 1, 2 and 3 billions of years. As the temperature at the surface of the earth is 0, it follows that the reciprocals of the computed temperatures represent the reciprocal gradients at that point.

Important among the results revealed by Table 3 are (1) that series (3) converges with sufficient rapidity as the 40th term is approached; and (2) the magnitudes of the reciprocal gradients are not inconsistent with the hypothesis of a homogeneous globe which has been cooling uniformly from a temperature of 76000°C. at its center through periods of 1, 2, or 3 billions of years.

The results of substituting in Table 2, Equation 3, the values of *a*, *b*, *c*, *d* obtained from the King-Barus melting point data are summarized in Table 5. The values for zero time are points on the adjusted King-Barus Curve. The tabular values for a depth of 1 kilometer are taken from Table 3.

TABLE 5

INITIAL TEMPERATURES AND TEMPERATURES (°C.) AT DIFFERENT DEPTHS FOR AN AGE OF 1, 2 AND 3 BILLION YEARS

Age of Earth Years	Depths from surface. Kilometers							
	1	50	100	200	300	400	500	600
0	1333.	1601	1894	2540	3261	4055	4920	5852
1×10^9	11.24	564	1124	2190	3163	4074	4976	5911
2×10^9	10.00	503	1010	2021	3014	3986	4950	5925
3×10^9	9.52	479	964	1944	2929	3915	4905	5908
	1000	1500	2000	3000	4000	5000	6000	6371 ($r=0$)
0	10195	16793	24351	40901	56951	69607	75975	76156
1×10^9	10213	16753	24254	40691	56631	69187	75547	75992
2×10^9	10230	16713	24157	40481	56312	68771	75115	75596
3×10^9	10243	16675	24062	40274	55996	68358	74677	75171

The tabulations reveal the following results:

1. As just explained the calculated temperatures at a depth of 1 kilometer are in agreement with the observed temperatures in the low temperature areas of the globe.

2. Cooling occurs at the center of the earth ($r=0$).

3. In a certain zone, 400–1000 kilometers, the temperatures rise for a time and then subside simulating in a way the flow of heat from a dike where the temperatures in the country rock first rise to a certain point after which they subside to their original values.

The first two conclusions cannot be questioned. The third conclusion implies that the temperatures given by the King-Barus melting point curve are either too low over the interval from 400 to 1000 kilometers (ab, Fig. 1), or, that the curvature of the King-Barus curve is such that it does not satisfy the requirements of a melting point curve. Equation 3, Tables 1 and 2, thus impose a condition on the melting point curve which must be fulfilled; namely, the cooling curve can not intersect the melting point curve at any point. This simple rule may serve as a guide in extending the melting point curve to great depths.

THE METHODS OF KELVIN, AND INGERSOLL AND ZOBEL

These methods are very convenient approximations to the rigorous solution of two special cases. Detailed tables for their use have been prepared by H. C. Spicer.[6]

To obtain the rigorous solution of these special cases from Table 2, it is merely necessary to put $b=c=d=o$ in the first case and $c=d=o$ in the second. Thus, multiplying each tabular value in the first column of Table 2 by the corresponding value of $e^{-(\kappa m^2\pi^2 t)/R^2}$ and taking the sum of the products we have the numbers designated (Σ) in Table 6. Multiplication of these numbers by the assumed values of (a) gives the temperatures for the depths indicated in the Table and for the time intervals of 1, 2 and 3 billion years.

For example, assuming a time interval of 2×10^9 years, and an initial temperature of 7000°F. (3871°C.) which is one of the values of (a) used by Lord Kelvin in his calculations, we have for the temperature at a depth of 50 kilometers: temperature $=0.132834\times3871=514$°C. The corresponding value given by Spicer in his tables is 541°C., which shows a difference of 27°C. between the approximate and rigorous calculations.

Again, assuming a melting point temperature of 1400°C., at the surface of the earth and a gradient of 4°C. per kilometer, we have for the initial temperature (v_m) at any point distant r from the center of the earth,

$$v_m = 26885 - 4r.$$

The computed initial temperature at a depth of 1000 kilometers is 5400°C. After the lapse of 2000 million years, the temperature at this point is reduced to 5339°C. as determined from Table 2 or to 5399°C. by the method of Ingersoll and Zobel.[6]

TABLE 6

KELVIN'S SOLUTION FOR SOME SPECIAL CASES
(κ=0.0064 C.G.S. units)

	1 kilometer			50 kilometers		
t years	1×10^9	2×10^9	3×10^9	1×10^9	2×10^9	3×10^9
Σ	.003813	.002651	.002135	.190104	.132834	.107204
a. °C.	10 000	10 000	10 000	10 000	10 000	10 000
v_1. °C.	38.1	26.5	21.3	1901	1328	1072
M. per °C.	26.2	37.7	46.8	26.3	37.6	46.6
Ft. per °F.	47.8	68.8	85.4	47.9	68.6	85.0
a. °C.	2000	2000	2000	2000	2000	2000
v_1. °C.	7.6	5.3	4.3	380.2	265.7	214.4
M. per °C.	131.1	188.6	234.2	131.5	188.2	233.2
Ft. per °F.	239.0	343.8	426.9	239.7	343.0	425.1

The computed reciprocal gradients in meters per degree Centigrade and feet per degree Fahrenheit in Table 6 show that the value of (a) falls between approximately 2000 and 10 000°C.; for reciprocal gradients of the order of magnitude of 240 feet per degree Fahrenheit have practically never been observed and values as small as 50 feet per degree Fahrenheit are not found in areas of undisturbed rocks.

It will be noted from Tables 5 and 6 that the reciprocal gradients for a particular age of the earth are almost constant up to a depth of 100 kilometers.

B. TEMPERATURES IN A RADIOACTIVE EARTH

Professor Daly[7] has emphasized the great importance of the problem of radioactive heat to the framer of an earth theory. Progress in the solution of the problem has been delayed, first, because of the absence of data on the distribution of radium throughout the interior of the earth; and, second, until recently, no one has given a rigorous treatment of the mathematical problem on the flow of heat from radioactive sources. Doctors Jeffreys[8] and Piggot[9] have summarized the data on the radioactivity of the rocks, with the result that the observations do not give us definite information on the concentration of radium in the surface layers of the earth or on the diminution of concentration with depth. Dr. Lowan's recent theoretical solution of

TABLE 7

COEFFICIENTS OF $\frac{A}{K}(1-e^{-(km^2\pi^2t)/R^2})\div 10^{10}$, $\alpha=0.00001$

m	$R-r$ Kilometers					
	1	50	100	200	1000	2000
1	0.00031 40	0.01582	0.03188	+0.06471	+0.35745	+0.77380
2	.00031 40	.01581	.03184	.06440	+ .31487	+ .42704
3	.00031 40	.01581	.03178	.06388	+ .25066	+ .05630
4	.00031 40	.01579	.03169	.06315	+ .17377	− .16691
5	0.00031 40	0.01578	0.03157	+0.06222	+0.09451	−0.18117
6	.00031 40	.01576	.03143	.06110	+ .02291	− .05536
7	.00031 40	.01574	.03126	.05979	− .03291	+ .07703
8	.00031 40	.01572	.03107	.05830	− .06792	+ .11591
9	.00031 40	.01569	.03085	.05663	− .08076	+ .05381
10	0.00031 40	0.01566	0.03061	+0.05481	−0.07372	−0.03927
11	.00031 39	.01563	.03035	.05283	− .05199	− .08343
12	.00031 39	.01559	.03006	.05071	− .02253	− .05169
13	.00031 39	.01555	.02975	.04846	+ .00736	+ .01793
14	.00031 39	.01551	.02942	.04610	+ .03134	+ .06269
15	0.00031 39	0.01546	0.02906	+0.04363	+0.04516	+0.04903
16	.00031 39	.01541	.02868	.04107	+ .04716	− .00411
17	.00031 39	.01536	.02828	.03844	+ .03836	− .04754
18	.00031 39	.01530	.02786	.03575	+ .02190	− .04590
19	.00031 39	.01525	.02742	.03301	+ .00223	− .00546
20	0.00031 39	0.01518	0.02697	+0.03025	−0.01598	+0.03558
21	.00031 39	.01512	.02649	.02746	− .02883	+ .04234
22	.00031 39	.01505	.02599	.02467	− .03395	+ .01227
23	.00031 39	.01498	.02548	.02190	− .03089	− .02571
24	.00031 39	.01491	.02495	.01915	− .02103	− .03844
25	0.00031 39	0.01483	0.02440	+0.01645	−0.00715	−0.01708
26	.00031 39	.01476	.02384	.01380	+ .00730	+ .01736
27	.00031 39	.01467	.02327	.01122	+ .01900	+ .03426
28	.00031 39	.01459	.02268	.00871	+ .02550	+ .02035
29	.00031 39	.01450	.02208	.00630	+ .02569	− .01021
30	0.00031 39	0.01441	0.02146	+0.00399	+0.01995	−0.02989
31	.00031 39	.01432	.02084	+ .00179	+ .00998	− .02237
32	.00031 39	.01423	.02021	− .00029	− .00167	+ .00410
33	.00031 39	.01413	.01956	.00224	− .01223	+ .02540
34	.00031 39	.01403	.01891	.00406	− .01934	+ .02335
35	0.00031 38	0.01392	0.01825	−0.00573	−0.02156	+0.00109
36	.00031 38	.01382	.01759	.00725	− .01867	− .02088
37	.00031 38	.01371	.01692	.00863	− .01161	− .02346
38	.00031 38	.01360	.01624	.00985	− .00222	− .00543
39	.00031 38	.01349	.01556	.01091	+ .00720	+ .01642
40	0.00031 38	0.01337	0.01488	−0.01182	+0.01447	+0.02282
1–40	0.01255 64	0.59826	1.02143	1.12390	0.98181	1.01457
41–m	0.63038 06	0.39915	− .04228	−0.08294	.....	.....
Σ	0.64293 70	0.99741	0.97915	1.04096	0.98181	1.01457
m	6740	2540	400	160	40	40

the problem of radioactive heat is more fortunate for it enables us to estimate some of the consequences of radioactive heat with a reasonable degree of certainty.

FORMULAS AND TABLE

Lowan's[10] equation for the rise in temperature (v_2) due to the generation of radioactive heat is

$$v_2 = \frac{A}{K}\cdot\frac{2R^2}{\pi}\cdot\frac{R}{r}\cdot\sum_1^\infty \sin\frac{m\pi r}{R}\left(\frac{(-1)^{m+1}}{m(R^2\alpha^2+m^2\pi^2)}\right)(1-e^{-(\kappa m^2\pi^2 t)/R^2}) \quad (5)$$

in which

A = rate of generation of radioactive heat at the surface of the earth

K = thermal conductivity

and α is defined by the equation

$$\phi = Ae^{-\alpha(R-r)} \quad (6)$$

which represents the rate at which heat is generated at depths $(R-r)$ from the surface of the earth.

The products of all of the factors in equation (5), except the first and the last, can be tabulated in a single factor for special values of α. Thus, equation (5) can be written,

$$v_2 = \frac{A}{K}\cdot\sum_1^\infty B_m(1-e^{-(\kappa m^2\pi^2 t)/R^2}). \quad (7)$$

Table 7 contains values of B_m for $\alpha = 0.00001$, and depths of 1, 50, 100, 200, 1000, and 2000 kilometers. The exponential factor reduces practically to unity for $t \geqq 1\times10^9$ years and $m \geqq 40$. Hence, two summations are necessary for large values of the time. In the summation of the first 40 terms, the exponential factor varies from 0 to 1, and in the second summation, the exponential factor is equal to 1 for each of the remaining terms of the series.

Let us assume that $t=\infty$. Then the exponential factor in (7) is always equal to 1, and the coefficient of A/K is $\sum_1^\infty B_m$. The first 40 values of B_m are given in Table 7 and as shown in the table, the sum of the first 40 terms in the second column is 0.01256, and the sum of all of the terms from $m=41$ to $m=6740$ inclusive, is 0.63038, or a total for the first 6740 terms of 0.64294, designated Σ in the table. Hence, the maximum temperature that can be obtained at a depth of 1 kilometer from a distribution of radioactive heat represented by

$\alpha = 0.00001$ in (5) is 0.64294 A/K. Similarly, under the same conditions, the maximum temperature that can be developed at a depth of 50 kilometers is 0.99741 A/K, and so on for the other depths.

The numbers designated Σ would be changed slightly by the addition of more values of B_m. For example, the value of B_m as tabulated in Table 7 for $m = 6740$ is 142×10^{-8}, but the sum of a large number of such terms would not affect the second or third decimal in

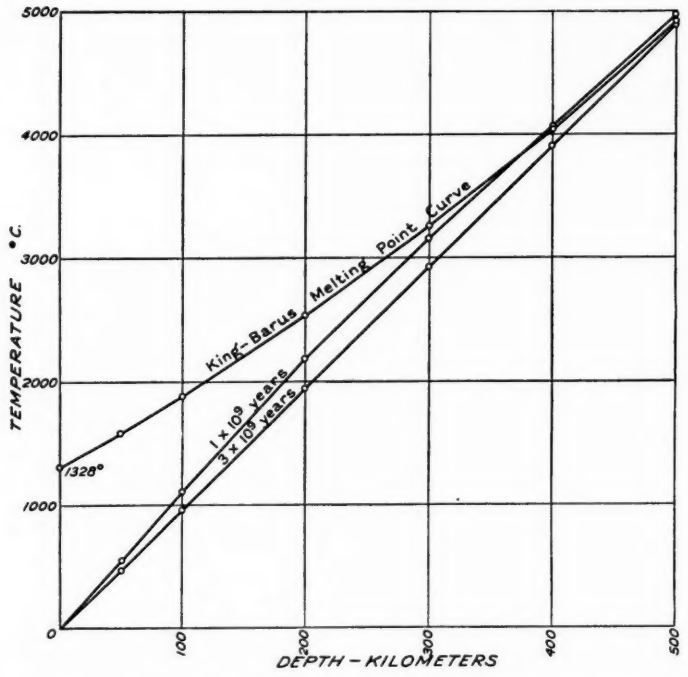

FIG. 2. Intersection of cooling curves with melting point curve.

the value of Σ. The slight irregularities in the remaining values of Σ show that a few more terms are needed to evaluate the function accurately in the first or second place of decimals for these particular depths.

SOME CALCULATED VALUES OF THE RISE (v_2) IN TEMPERATURE

In Table 8, the coefficients of A/K are summarized for $t = \infty$ and $t = 1, 2, 3 \times 10^9$ years; and $\alpha = 1 \times 10^{-5}$. The calculated temperatures are based on the values, $A = 1 \times 10^{-12}$ calories per cc. per sec.;

TABLE 8

COEFFICIENTS OF A/K FOR RADIOACTIVE HEAT; AND RESULTING TEMPERATURES.
$A/K = 2.22 \cdots \times 10^{-10}$, $\alpha = 0.00001$

t	∞	1000	2000	3000
1 kilometer				
Coefficeint	0.64294	0.63912	0.64029	0.64080
v_2	1.42878	1.42027	1.42287	1.42400
v_1	0.00000	11.23938	9.99986	9.52154
v	1.42878	12.65965	11.42273	10.94554
$1/b$ M.°C.	699.90	78.991	87.545	91.361
$1/b$ Ft.°F.	1276.	144.0	159.6	166.5
50 kilometers				
Coefficient	0.99741	0.80732	0.86459	0.89022
v_2	2.22	1.79	1.92	1.98
100 kilometers				
Coefficient	0.97915	0.60782	0.71568	0.76540
v_2	2.18	1.35	1.59	1.70
200 kilometers				
Coefficient	1.04096	0.37103	0.53825	0.62492
v_2	2.31	0.82	1.20	1.39
1000 kilometers				
Coefficient	0.98181	0.0	0.0	0.013
v_2	2.18	0.0	0.0	0.03
2000 kilometers				
Coefficient	1.01457	0.0	0.0	0.0
v_2	2.25	0.0	0.0	0.0

$K = 0.0\ 45$; or $A/K = 2.22 \cdots \times 10^{-10}$; and $\kappa = 0.0064$ C.G.S. units. The computed temperature distributions for 1 and 3 billion years, and for infinite time, are shown in Fig. 3. The dotted segments of the curves have been sketched. They are not based on calculated data.

The curve for $t = \infty$ represents the temperature distribution for a steady state, that is, the rate of escape of the heat at the surface of the earth equals the rate at which heat is being generated in the interior of the earth. This curve which is typical in form for all values of the thermal constants represents the maximum temperature that can be obtained at any given depth as a result of radioactive heat.

The temperatures cease to rise when this state is reached. For finite time intervals, we have, as illustrations, the respective curves for 1 and 3 billion years, Fig. 3. These curves pass through a maximum and are asymptotic to the depth axis. They practically vanish for depths of approximately 1000 kilometers or less as shown in Fig. 3 and Table 8.

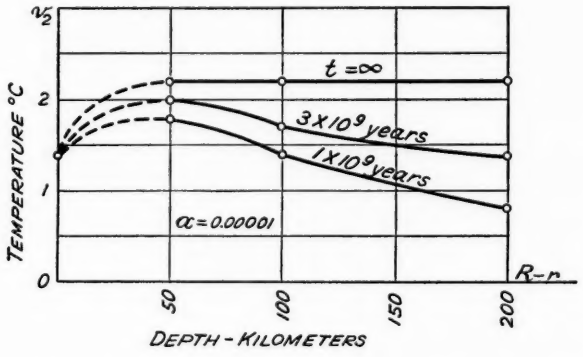

FIG. 3. Temperatures (v_2) due to heat of radioactivity.

SOME SPECIAL APPLICATIONS OF THE TABLES

Table 1, 2, 7, and 8 enable the investigator to test numerous hypotheses in regard to the origin and distribution of the temperatures in the interior of the earth. Table 7 should be extended to include $\alpha=0.000\ 001$ and $\alpha=0.000\ 0001$.

In order to apply the tables to specific problems, it is necessary to know the values of A, K, and α. Assuming that the temperature gradient is known, the value of A/K can be determined when α is known, or conversely, α can be determined when A/K is known.

Laboratory determinations of A and K do not agree very closely. Professor Daly[11] gives $K=0.0051$, 0.0052, and 0.0061 C.G.S. units for three specimens of granites; and $K=0.0035$, 0.0040, and 0.0041 for three specimens of basalts. Thermal conductivities and diffusivities vary slightly with temperature and pressure. Professor Joly[12] gives $A=1.15\times10^{-12}$ for rocks of granitic character and Dr. Jeffreys[13] gives $A=1.3\times10^{-12}$ for granite; 0.83×10^{-12} for diorite; and 0.50×10^{-12} for basalt. Using recent data on the abundance of uranium, thorium, and potassium, Dr. R. C. Wells,[14] Chief Chemist, U. S. Geological Survey, finds $A=0.80\times10^{-12}$ for granite and $A=0.21=10^{-12}$

for basalt. With so few values at our disposal, it will be convenient and sufficiently representative of the data to put $A = 1 \times 10^{-12}$ calories per cc. per second in the calculations which are to follow.

As the true value of α can not be determined from existing data, it must suffice for the present to determine possible upper and lower limits of this quantity.

Let it be assumed therefore that radium is the source of all of the heat that escapes from the surface of the earth. The values of v_2 for 1 kilometer in Table 8 show that $\alpha < 0.00001$ for any value of A/K that is likely to arise. Thus $\alpha = 0.00001$ is an upper limit.

To find the lower limit, put $\alpha = 0.000\ 001$. The coefficient of A/K in this case is approximately 9.14×10^{10}. Assuming as before that $A/K = 2.22 \times 10^{-10}$, the corresponding reciprocal gradient determined from the temperature at a depth of 1 kilometer is 1°C. in 49.23 meters (1°F. in 89.7 feet). As observations show that the reciprocal gradients in undisturbed areas are between 1°F. in 100 and 200 feet[15] (1°C. in 54.9 and 109.7 meters), it follows that $\alpha > 0.000\ 001$ unless $A/K < 2.22 \times 10^{-10}$. That is, the true value of α is a little larger or a little smaller than 0.000 001. The value, $\alpha = 0.000\ 0001$, is certainly a lower limit.

Now that α is known to fall between the limits 0.00001 and 0.000 0001, it is possible to estimate from Formula 5 the rate at which heat is being generated at given depths. Table 9 gives the factors (F) by which (A) is to be multiplied to obtain the required rate (AF).

TABLE 9

DIMINUTION OF RADIOACTIVE HEAT WITH DEPTH. VALUES OF F.

$R-r$	α		
Kilometers	0.00001	0.000 001	0.000 0001
10	0.00005	0.36788	0.90484
20	.00000	.13534	.81873
30	.00000	.04979	.74082
40	.00000	.01832	.67032
50	.00000	.00674	.06653

Table 9 shows that a distribution of radioactive heat represented by $\alpha = 0.00001$ implies that practically all of the radium is contained in an outer shell which has a thickness of 10 kilometers. For the next value, $\alpha = 0.000\ 001$, and $A = 1 \times 10^{-12}$, the rate of generation of heat at a depth of 50 kilometers is about 7×10^{-15} calories per cc. per second.

Under these conditions, most of the radioactive heat is developed at depths of much less than 50 kilometers. The other value, $\alpha = 0.000\ 0001$, is an extreme case and need not be considered.

Let us now proceed to determine the maximum temperature that can arise from radioactivity. According to Table 8, Fig. 3, the maximum rise in temperature (v_2) resulting from radioactive heat under the condition, $\alpha = 0.00001$, is very small. Lowan[10] gives the following figures:

$K = 0.007\ A/K = 1.43 \times 10^{-10}\ \alpha = 0.000\ 00066$ Max = 440°C.

$K = 0.005\ A/K = 2.00 \times 10^{-10}\ \alpha = 0.000\ 00068$ Max = 370°C.

The maxima have been taken from Lowan's Fig. 1. As his values of α are near the lower limit, it may be assumed with a reasonable degree of certainty that the total contribution of radioactivity to the heat of the earth does not produce a maximum rise in temperature of more than 500°C., and, as shown on Lowan's Fig. 1, the maximum occurs at a depth of about 30 kilometers. This is probably the best estimate that can be made until the curves for $\alpha = 0.000\ 001$ and $\alpha = 0.000\ 0001$ are calculated.

To test the possibility of melting at a depth of about 30 kilometers, we find from Table 5, Fig. 2, an initial melting point temperature (v_m) of 1490°C. which is 990°C. above the estimated maximum of 500°C. at that depth. These calculations imply that radioactivity alone may not suffice to induce melting at moderate depths. However, the fact that the maximum temperature occurs at depths at which lava flows are supposed to originate, suggests that the point at which $v = v_1 + v_2$ makes the nearest approach to the melting point curve is related in some way to the cause of the flows. The additional heat that seems to be necessary may thus have been supplied by gravitational contraction, uplift and erosion, or other geodynamical processes.

A different condition exists at depths of about 500 kilometers—the region of deep-focus earthquakes. Here the addition of a few degrees to the cooling curves, Fig. 2, may suffice to bring the temperatures up to the melting point; for, although the rise in temperature (v_2) is small, cooling of the rocks through long intervals of time is also small such that the net result of the two activities may be a melting point temperature at that depth.

Intermediate between the depths of 30 and 500 kilometers, the combined effects of heating and cooling may result in volume changes through recrystallization of substances that are near their critical

temperatures. Whether or not these expansions and contractions are the cause of earthquakes is not known. Prof. E. W. Brown[16] states that a change in volume of 1 per cent in a layer with a thickness of 1 mile at a depth of about 50 kilometers would be sufficient to explain the observed changes in the length of the day. He further states that the hypothesis provides a mechanism for mountain building.

CONCLUDING REMARKS

Complete tables have been calculated for use in evaluating the temperatures in a non-radioactive earth. Tables sufficient for the purpose of obtaining a general idea of the behavior of radioactive heat are also given in this paper. All of the fundamental tables (1, 2, 7, 8) are based on rigorous mathematical theory.

The assumption of a homogeneous earth (κ=constant) is, of course, incorrect. Nevertheless, the results of our calculations suggest certain important possibilities. Heretofore, for example, it has been assumed that cooling did not extend below 1000 kilometers. This conclusion is based on the rather dubious assumption that the initial melting point curve is a straight line parallel to the depth axis ($b=c=d=o$ in Tables 1, 2, 6). A melting point curve in which the temperatures increase with the depth would seem to be more in accordance with the facts although the temperature at the center of the earth may not reach the high value of 76000°C. which was found to be consistent with the low temperature gradients observed in areas that have remained practically undisturbed through very long intervals of time.[17]

Temperatures arising from radioactive heat are limited because of the fact that they constantly approach a steady state ($t=\infty$, Fig. 3). When the steady state is reached, the rate of escape at the earth's surface is equal to the rate of generation of heat from radioactive sources within the earth, and a further accumulation of heat is impossible.

Ages of the earth obtained by means of the geothermal equations (3, 5) are consistent with those obtained by the method of atomic disintegration (see Table 8). The equations possess further possibilities in that, to a certain extent, they determine the trend of the melting point curve at all depths; and, with reference to such problems as the causes and depths of lava flows and deep-focus earthquakes, the information given by the geothermal equations may be indispensable as a means of obtaining the correct solutions of the problem.

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GEOPHYSICAL EDUCATION IN A DEPARTMENT OF GEOPHYSICS*

JAMES B. MACELWANE, S.J.†

There have been a number of valuable discussions on the subject of geophysical education. Among these is the recent symposium on geophysical education which was sponsored by the A.I.M.M.E. However, the impression made upon one by the study of the various papers published on the subject of geophysical education is a sense of incompleteness, a feeling that the sight of the forest is lost because of preoccupation with the trees.

In order to view the subject of geophysical education as a whole it is necessary to seek a vantage point from which the multitudinous sections of the field will appear in their correct perspective.

It has been said that geophysics is the physics of the earth. Geophysics has been described as a marriage of physics and geology. But geophysics is not physics. Neither is geophysics a branch of geology. Much less is it a mere tool of the geologist. Geophysics is a science in its own right with its own complex subject matter, its own methods of investigation and its own point of view. As a science it has many branches, for example: geodesy, seismology, meteorology, terrestrial magnestism and electricity, physical oceanography, volcanology and hydrology, all of which are recognized as sections in the American Geophysical Union. Diverse as are these branches of geophysics they are not divergent. They all deal with the interlocking complex physical phenomena of this earth of ours. They are all observational rather than experimental in character. The physicist in his laboratory has the point of view of an experimenter asking questions of an enclosed system completely controlled. He calls his witnesses one at a time. He makes them answer not only individually but independently. The geophysicist takes over the equipment, the technique and the results of the physicist and applies them directly to nature. His witnesses are not segregated nor called individually, nor cross questioned in solitude, as it were, in the privacy of a closed system. They cannot be thus isolated without disturbing the environment in which they play

* Paper presented at the Annual Meeting of the Society of Exploration Geophysicists, March 21–23, 1939.

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their natural roles. The geophysicist must take them unawares, as it were, as they move about their daily tasks. Armed with his candid camera and his dictograph, he collects his evidence, sifts it, analyses it and draws his conclusion. The geophysicist cannot be satisfied with the methods and the equipment of the physicist alone for these would uncover for him only the activities of individuals outside their natural milieu. As a geophysicist he must investigate also the undisturbed environment and for that purpose he must draw upon the other earth sciences and especially upon geology. The geologist, too, is an observational scientist. He goes directly to nature, he observes the processes that are now at work on the earth's surface in so far as these tend to build up, tear down and otherwise modify land forms and thus bring into being new rocks to serve as constituent parts of the solid earth. He studies in minute detail the rock types already present in the earth's crust and correlates them with the end products of the processes now going on at the earth's surface. Step by step he builds up multiple hypotheses connecting present day processes and their end results with the rocks and minerals of the past and the processes which might have contributed to their formation or deposition. Shrewdly he lays his traps for pertinent facts. With quiet persistence he goes over the ground hunting for decisive factors which he can use to eliminate one by one the hypotheses he has constructed until by patient comparison he can concentrate the probabilities on the one or the other. Thus the geologist is enabled to draw reliable conclusions as to the constitution and structure of the earth's crust and of the deeper interior; thus also he proceeds to locate the useful minerals which may be exploited economically. The geophysicist and the geologist are on exactly the same footing when looking into the depths of the earth. Both must observe from the surface. The difference between them lies partly in the phenomena that are observed by each, partly in the methods and instruments with which they carry on their exploration but partly also and perhaps principally in their habitual attitude of mind and in the background of specialized training with which they approach the same problems.

The background of the typical geologist will be mineralogical, physiographical, structural, stratigraphical, frequently petrographical and paleontological but it will seldom be physical and mathematical. The geophysicist, on the other hand, while sharing the background of the geologist, will be a mathematician, and, in addition, will have all the background of the physicist. President Hotchkiss said,

"A friend of mine made the remark that to be a real geophysicist a man should have a doctor's degree in mathematics, another one in physics, and another one in geology; merely having a doctor's degree in geology would not do."[1] While this ideal of the geophysicist was intended to be somewhat facetious and scarcely attainable in practice the underlying thought is very sound and I think it is from this point of view that we should consider geophysical education. In the writer's opinion it is impossible to make a real geophysicist in an undergraduate curriculum. It is difficult enough to make one with the minimum training for the doctorate even if we emphasize in his training only a few of the geological, geophysical and physical fields in which his own major subject is ultimately rooted and which he needs in order to become a really independent investigator.

The members of this Society are more immediately interested in those fields of geophysics which are or can be applied in geophysical prospecting. Hence for the purposes of this discussion we may narrow our view to the fields of geodesy, seismology, volcanology, terrestrial magnetism and electricity and radioactivity; and we may consider in relation to these fields only those students who intend to become professional geophysicists in counter distinction to students in other fields such as geology, or mining, or petroleum engineering who are interested in knowing something about geophysics. There is a well marked distinction between the chemist and the chemical engineer and between the physicist and the mechanical or electrical engineer. In like manner there should be a clear distinction between the geophysicist and the geophysical engineer. The engineer in any field is expected to receive a solid fundamental training in the sciences which play a part in his future work but his training is always oriented toward the practical application of principles and results. The so-called pure scientist, on the other hand, is occupied with the fundamental research whereby these principles and results are derived. In exploratory geophysics there is a place for both types of training. As Doctor Barton has well said, "The rank and file, composing perhaps two-thirds of the personnel of exploratory geophysics, is not, and should not be, composed of highly trained men. The routine, stereotyped character of the work and the impossibility of promotion of more than a few from these lower ranks even under the best circumstances justify companies in filling the bulk of these rank and file positions with men having only a bachelor's degree at the most."[2]

[1] A.I.M.M.E. *Geophysical Prospecting*, 1932, p. 198.
[2] A.I.M.M.E., *Technical Publication* No. 950, V, p. 8.

Again, "The future for a man having a Ph.D. in geological geophysics will be as good as for a Ph.D. in geology. Geophysical prospecting is weak in men who have combined good training in geology, in physics and in mathematics. As such a man is both geologist and geophysicist, he is eligible for promotion to geological as well as geophysical positions. As the reduction in domestic exploratory geophysics takes place he will be able to transfer to geology without loss of rank. He will have a reasonably good chance for promotion to the highest executive positions. Men whose duties bring them frequently into conferences in regard to important company problems and policies have the best chance of promotion to those positions. The geologist-geophysicist's duties bring him into frequent conferences in planning geophysical programs and in regard to activities as result of the discovery of geophysical prospects. His chance of promotion, therefore, is much better than that of the physicist-geophysicist who rarely gets into a conference except with his immediate chief or the chief geophysicist in regard to his own narrow problem.

"For the Ph.D. in physics-geophysics, the future is fairly good, but is more restricted than for the Ph.D. in geological geophysics. Petroleum engineering and probably refinery engineering also offer considerable field of activity for him."[3]

With most of Doctor Barton's statements the writer would agree; but there is one point to which he takes exception. Is there such a thing as a physicist-geophysicist or a geologist-geophysicist? I believe these are only euphemisms for lack of adequate training as a geophysicist. The qualifications of both of these types should be combined in the one person and it should not be by way of distinct and separate training in geology and in physics. Built upon a solid foundation of physics, mathematics and geology we must rear the edifice of geophysical education; for geophysics, as we have said, is neither physics nor geology but is a distinct science in its own right. Therefore, while I sympathize with the point of view of Doctor Barton when he says, "I am in hearty disagreement with the recommendation that geophysics should be done by cooperative work between a geologist and a geophysicist. That arrangement would be comparable to doing petrography by having a physicist at the microscope and a geologist sitting by his side. The interpretation of the geological significance of geophysical data can be done efficiently only by a physicist who has become also a good geologist or by a geologist who is also a good geophysicist."[4] I differ with him in that he does not go far enough. The

[3] *Ibid.*, p. 3.

[4] *Ibid.*, p. 8.

interpretation of the true significance, geological and physical, of geophysical data can be safely entrusted only to a thoroughly trained geophysicist.

The training of the geophysical engineer will be treated by Professor Wantland in the next paper. Hence we shall focus our attention principally on the making of a geophysicist in a university department of geophysics.

In France there are five institutes of geophysics which form integral parts respectively of the Universities of Paris, Strasbourg, Clermont-Ferrand, Toulouse and Algiers. Of these only two—those at Paris and Strasbourg—are departments of instruction in the strict sense. The Geophysical Institute at Strasbourg was founded in 1919 and that at Paris in 1921. Each of these two is a unit or section of the Faculty of Science. Only one—the University of Strasbourg—has organized a special practical curriculum leading to the diploma of geophysical engineer. The Director of the Geophysical Institute at Paris, Doctor Charles Maurain, was kind enough to write me and to send literature describing the studies taken by their students. The first step on entering the university, after completing a good undergraduate course and receiving the baccalaureate in the lycée, is to qualify for the degree of licentiate in science. To do this it is necessary to obtain three—and beginning with the next year, four—certificates of higher studies. The subjects in which certificates are granted are arranged in cognate groups. Each of the three or four certificates secured must belong to a different group. For each of these certificates there is required a written examination, an oral examination and a test on prescribed laboratory work. The certificate in geophysics covers the constitution of the earth, seismology, terrestrial magnetism and electricity, meteorology, aerology, radiation, ionization of the atmosphere and transmission of electromagnetic waves in the atmosphere and in the solid earth. The laboratory includes practical work with all the usual instruments employed in these various fields. Visits are prescribed to meteorological, magnetic and seismological stations. The next step in the university is the diploma of higher studies, and finally the doctorate. Doctor Maurain writes, "You would undoubtedly like to ask me what conditions I consider desirable for a person who wishes to aspire to the direction of geophysical work. My feeling is that the desirable prerequisites are a thorough training in mathematics, physics and geology."[5]

[5] Private communication.

The student guide book of the Faculty of Science of the University of Strasbourg states that the normal duration of studies for the degree of geophysical engineer is two years after the undergraduate bachelor's degree together with the second part of the curriculum in mathematics, or after the licentiate in science. Besides the theoretical training, it includes practical field work in the Vosges Mountains with the magnetical, electrical, gravimetrical, seismological, radiological and thermic methods of prospecting. Doctor Edmund Rothé, founder and director of the Institute of Geophysics at the University of Strasbourg and formerly Dean of the Faculty of Science wrote me concerning the field camp at Sainte-Odile in the Vosges and the course of study for the degree of geophysical engineer and added, " . . . but all this concerns only the preparation of ordinary students who are to become geophysical engineers and enter the industry to do geophysical prospecting. As regards those who seek the doctorate with the intention of becoming candidates for professional chairs in a University the preparation should be very much more complete. The best way will be to indicate what my son has done in order to qualify for this career. In the first place he made one year of studies in the faculty of letters where he secured the certificate of geography which is indispensible in my opinion to one destined for geophysics. Next he transferred to the faculty of science where he obtained successively the certificates of general mathematics, general physics and general chemistry. Then he secured the diploma of higher studies in physics his thesis being concerned with the magnetic variometer. Only then did he devote himself completely to geophysics, taking his certificate in geophysics and then a certificate in geology. Such a very general preparation renders one well qualified to follow studies in geophysics and in particular to pursue the study of seismology. Fourteen months passed in the Polar regions with the French expedition to Scoresby-Sund permitted him to reduce his knowledge to practice and on his return he completed his research and undertook to present and defend his doctoral dissertation. Such, my dear colleague, is my idea of the foundation of very solid general culture which a future professor should possess and this is the advice I shall give to anyone destined for geophysics."[6]

Geophysical education in Germany goes back to the beginning of the century. Doctor Sherwin F. Kelly in sketching the history of geophysics remarks, "Yet, old as is this marriage of geology and physics,

[6] Private communication.

it was not recognized as a department of knowledge worthy of distinctive appellation until nearly 1853. In that year the word appears for the first time in a lexicon published in Germany. Thirty-five years later it is found in English, in a review of a geology textbook. The reviewer applies it to problems concerned with the earth's crust and the primitive state of the earth; wherein, of course geophysics borders on cosmic physics, and the origin of the solar system and of the planet whereon we dwell. The first publication definitely devoted to the physics of the earth was Gerland's *Beiträge zur Geophysik*, which appeared at the University of Strasbourg in 1887, and has been followed since by a regular series of volumes."[7]

The Institute for Geophysics at the University of Göttingen had its beginning in the Observatory of Terrestrial Magnetism which was directed by Gauss and Weber from 1832 to 1868. In 1898 Emil Wiechert was appointed Professor of Geophysics and in the fall of 1901 he occupied the new buildings of the Geophysical Institute on the Hainberg. The work of the Göttingen Institute has influenced geophysics not only in Germany but throughout the world. Many of those who have left its halls with the doctorate in geophysics have become outstanding leaders in the science. There are a number of other geophysical institutes in Germany, such as Königsberg and Potsdam and notably that of Leipzig under Weickmann and the National Institute for Earthquake Research at Jena which are very active. Connected with the Prussian Geological Survey in Berlin there is a well staffed Institute for Applied Geophysics. However, curricula for geophysical engineers do not seem to have been organized at any of the German institutions.

In Italy there are a number of geophysical institutes connected with universities and colleges but they are rather for purposes of geophysical observation and research than for advanced instruction. One of the outstanding institutes is undoubtedly the Reale Istituto di Geofisica in Trieste under the directorship of Doctor Vercelli. The geophysical set-up in Italy has been reorganized recently under the National Research Council. The central administrative institution in Rome is now called the Istituto Nazionale di Geofisica.

The principal centers of geophysics in England would seem to be London and Cambridge. The eminent leader and professor of geophysics at Cambridge, Doctor Harold Jeffreys, has devoted his attention largely to the broad fundamental problems concerning the

[7] A.I.M.M.E., *Technical Publication* No. 950, I, p. 10.

physics of the earth, meteorology, seismology and earth structure. On the other hand Doctors H. Shaw and Lancaster-Jones, and the Science Museum at South Kensington have been much more closely associated with geophysical prospecting.

In Japan instruction in geophysics is given in the Seismological Institute and in the College of Engineering of the University of Tokyo. Advanced study and research in seismology is carried on under the auspices of the Earthquake Research Institute of the Tokyo Imperial University and of the National Research Council, the Central Meteorological Observatory and the several geophysical institutes of other universities.

In Canada the three centers of geophysical research and instruction are the Dominion Observatory at Ottawa, the University of Toronto and McGill University in Montreal. In both of the Universities the geophysics is too closely subordinated to the Department of Physics and has too little association with geology.

In the United States geophysical education is of comparatively recent origin; and while it is expanding rapidly in our universities and engineering schools it is usually a stepchild of geology or physics. In the University of California studies in meteorology are under the department of geography, those in seismology, both undergraduate and graduate, are under the Department of Geological Sciences. At California Institute of Technology geophysics is one of the fields or sections in the Balch Graduate School of Geological Sciences. Much excellent work in geophysics is being accomplished at Harvard University and at Massachusetts Institute of Technology but in neither place is the geophysics separately organized as a department or even unified under one head.

The second department of geophysics in the United States was organized in 1928 at the Colorado School of Mines in Golden, not only as an undergraduate engineering department but also as a graduate department. I shall leave to Professor Wantland the description of its curriculum.*

The first department of geophysics in the United States was established in the spring of 1925 at Saint Louis University and the writer was called from the University of California to become its Director. It grew out of the Geophysical Observatory founded by Father J. B. Goesse, S.J., in 1909. For several years after the organization of the department in 1925 the emphasis was placed on graduate

* Published as a companion paper, see p. 91.

training for the Master's and Doctor's degrees. These graduate studies were based to a much greater extent on solid undergraduate training in physics, mathematics and geology than on undergraduate courses in geophysics because it was the conviction of the writer that seven years is all too short a time to train a geophysicist and that a geophysicist will succeed in proportion as his training in physics, mathematics and geology has been broad and thorough before he is called upon to integrate his knowledge of these subjects into the special field of geophysics. The writer's conviction has not changed with the years; but the general demand for undergraduates with some training in geophysics has moved him to organize an undergraduate curriculum in which the first two years are devoted to general education and to fundamental courses in physics, chemistry, mathematics and geology. In the third and fourth college years a field of concentration is organized in which the student takes a year course in petrology, and semester courses in structural geology, economic geology and field geology and five semester courses in applied geophysics—one in the fundamental mechanics, mathematics and physics of the various fields and one semester course each in seismic, magnetic, gravitational, and electrical methods of prospecting with emphasis on the theory that underlies each method and with appropriate laboratory and field exercises. Little time is spent in training the student in the detailed technique of prospecting. Some of the reasons for emphasis on the science of geophysical prospecting rather than on the practice of its techniques are these:—First, the technique differs from one company to another; secondly, this technique changes rapidly with circumstances; thirdly, and principally, because the student with a thorough theoretical background may be expected to master any technique and adapt himself to it much more readily than if he learned it as a rule of thumb.

I am quite in accord with the statement of Doctor Blau, "I might say one more word about field work for students in colleges. I am not a great admirer of engineering schools of the old kind. It is more important, in my opinion, to learn what makes the motor go than it is to find out where the switch is. We can tell a man where the switch is if he has already learned what makes the thing turn around.

I do not care much for students that have been trained by somebody else in the use of a geophysical methods as that person or that instructor thought the thing was used five years ago. He does not know how we are using it now and we are not telling him just now

either. But I would like the instructor to give the student a good training in the fundamentals, and if the latter knows potential functions; if he knows something about gravitation, if he knows something about the earth's magnetic field, then, in one or two months he can be trained to use a torsion balance or a magnetometer. As a matter of fact, when we get into a pinch, we can train a man who has had only a high school education to use a magnetometer or a gravity meter. I think, therefore, that a student is wasting his time when learning to operate a magnetometer according to the formula of some person that cannot possibly know how we are using it or how he is going to have to use it after he gets out of college."[8]

The department of Geophysics at Saint Louis University works in unusually close association with the departments of geology and physics. Looking to the future in the light of past experience it would seem that the geophysical engineer has a definite place and that he will hold it the more readily the better his training in geophysical engineering. As Professors Heiland and Wantland have well said: "The industry employing geophysical engineers has a right to expect them to be trained in *geophysics*, not merely in geology, physics and mathematics. An oil company hiring a petroleum engineer today expects him to be familiar with methods of petroleum production and refining and does not hire a man for such a job if his education has been confined to chemistry or basic engineering only. Training in details of methods applied by the company will always be necessary, especially for men just out of school, and is common practice in other fields of engineering."[9] On the other hand, the demand for real geophysicists with thorough training at least as far as the doctorate has always been and is now far in excess of the supply. In the words of Doctor Barton, "Geophysicists are of many grades; not all who term themselves, or are termed, geophysicists, are such, and not all men with long experience in geophysics are competent interpreters. In making use of geophysics, the oil prospector will be wise to ascertain what sort of geophysicist the man is whom he would employ. The most competent geophysicist, like the geologist, is not omniscient, but in general he is well aware of the difficulties and uncertainties of geophysics and of the inadequacy of his knowledge. A competent physicist is not necessarily a competent geophysicist or geophysical interpreter, and his physically plausible interpretation may be highly im-

[8] A.I.M.M.E., *Technical Publication* No. 950, discussion, pp. 1–2.

[9] A.I.M.M.E., *Technical Publication* No. 950, III, p. 4.

probable or impossible geophysically and geologically. A geologist who has had a superficial contact with geophysics may be misled by the seductive resemblance of magnetic and gravitational isogams to structure contours, and may make an interpretation of magnetic or gravity survey which is plausible geologically but wholly impossible geophysically."[10]

And may I also quote Doctor Slichter, "I should like to add a brief remark about the need for more well trained young geophysicists. It has been our experience that not enough young men are preparing themselves for responsible jobs in this field. Certainly we have always had difficulty in finding enough good men to fill good positions available; this shortage of suitable candidates persisted even during the 1932 depression, and seems to be still with us. I am speaking, of course, only about well trained men, with several years of graduate work."[11]

In closing I should like to make a strong plea for the organization of all geophysical education under distinct departments of geophysics manned by trained geophysicists both in our universities and in our engineering schools; for it is only in a geophysical atmosphere that the distinctive viewpoints and methods of geophysical science can be expected to bear their full fruit.

[10] *The Science of Petroleum.* Published by the Oxford University Press, p. 326.
[11] A.I.M.M.E., *Technical Publication* No. 950, VI, discussion, pp. 3–5.

GEOPHYSICS EDUCATION AT AN ENGINEERING INSTITUTION*

DART WANTLAND†

ABSTRACT

The background and development of the courses of instruction in Exploration Geophysics at the Colorado School of Mines as well as the limiting factors in shaping a geophysics curriculum and the time devoted to various phases of the graduate courses in applied Geophysics offered are discussed. The present positions occupied by geophysics graduates of the institution after 12 years of operation of the Geophysics Department are shown, and a summary is given of the replies of these men to an inquiry as to their opinion of the instruction they received.

INTRODUCTION

In discussing Geophysics Education, I will endeavor to set forth, first, the purposes for which the Department of Geophysics at the Colorado School of Mines was established, second, the development of the program of instruction in applied Geophysics offered there, and third, the summarized reactions of our graduates in response to a recent questionnaire asking their opinion of the training they received with us in the light of their subsequent professional work.

THE FIRST GEOPHYSICAL COURSES AT THE COLORADO SCHOOL OF MINES

Mr. Max W. Ball was the first to call to the attention of the members of the American Association of Petroleum Geologists[1] that a Department of Geophysics was being established at the Colorado School of Mines. Mr. Ball stated as follows: "The intention is to carry on research work in the application of Geophysical Methods to the development of mineral deposits, to train men for practical geophysical work in the oil and mining industries, and to inspire picked men to carry on further research work in industry and in educational institutions." He added that it was felt " . . . that a real service can be rendered the industry by educating men in geophysical principles, giving them preliminary training in geophysical methods, and above all, developing the research spirit in the application of these methods to the problems at hand." The preliminary organization work was

* Paper read at the ninth Annual Meeting of the Society of Exploration Geophysicists, Oklahoma City, Mar. 23, 1939.

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[1] A. A. P. G. Bull., Vol. 10, No. 12, Dec., 1926.

completed late in 1926; two geophysics courses were offered in the second semester of the academic year 1926–27; instruction was begun in January, 1927.

Fig. 1 is a photograph of the class in the first graduate course in Geophysics at the Colorado School of Mines which was on Torsion Balance Prospecting. The members of this group were, starting on the left, Mr. Jas. E. Dick, Dr. J. A. Malkovsky of the late Czechoslovakia, Mr. G. H. Westby, Mr. E. W. Pingstaff, the writer whose head is just visible, and Dr. C. A. Heiland, our professor. Mr. Th. Zatterstrom and Mr. H. A. Aurand were also members of this class.

FIG. 1. The First Class in Graduate Geophysics at the Colorado School of Mines. Spring of 1927. From left to right: J. E. Dick, Dr. J. A. Malkovsky, G. H. Westby, E. W. Pingstaff, Dart Wantland, and Dr. C. A. Heiland. Mr. Th. Zatterstrom and Mr. H. A. Aurand were also members of this class.

The initial graduate course which established standards for those to follow, dealt with the theory and application of the torsion balance. As a member of the class I can attest the rigorousness of the course; we labored arduously through a thorough development of gravitational potential theory and the derivation of the basic Eötvös torsion balance formulae. As the saying goes, we members of this first class were "practical men"; while technically trained, our interest at that time was more along the lines of learning how to operate the instrument in the field and how to interpret the results, particularly the

latter. Some of us wondered at times, just where these mathematical derivations which were pounded home by our professor fitted in. The theoretical study was, however, supplemented by laboratory and field work; we set stations and, toward the last of the course, were given instruction in interpretation. It was not until some time later that, as individuals, we began to realize that after all, we had been well instructed. It was soon learned in practice that while field work was largely routine, with the thorough groundwork in the fundamentals of the method, pertaining to the functioning of the instrument, the forces measured and the effects involved and their relation to geology, we had in fact, the tools for the proper interpretation of results.

I have given the reactions of the members of this class in the initial graduate course, as it brings out the importance of basic theory in a geophysics course. Moreover, in 1927 we had not been allowed to over-specialize, a policy which has been continued, for we were given a 3-hour lecture course in General Geophysics which was a blend of pure and applied geophysics. This taught us the scope of the subject as a whole and in a broad way, the geologic background, basic principles, and field of application of the four major prospecting methods.

If we look back on the state of geophysical exploration in the United States in the spring of 1927 it can be appreciated just how little published information on this subject was available to American geologists. The translation of Ambronn's book did not appear until 1928, and it was followed by that of Eve and Keys in 1929. At that time the stream of truly informative papers on geophysics had not yet begun; in January of 1927, geophysical exploration for petroleum in the United States was just 3 years old (if the fall of 1923,* with the location of the Nash salt dome, can be marked as its recognition date).[2] There were then definitely no guides and rules on geophysics education.

In establishing the point of view from which this article is presented, may I say that Geophysics as here used means Applied Geophysics and not Earth Physics. Naturally, we are not unmindful of the relation of the broader aspects of Earth Physics as the background of the applied branches and such material is used in bringing out basic relationships and showing the development of procedures

* It is realized that certain experiments and mining geophysics antedate 1923 in the United States.

[2] Barton, D. C., A.I.M.E. G. P. Volume, 1929, p. 444.

and equipment in building our graduate courses. In fact, until 1932, we taught a general course of one semester in Pure Geophysics which was discontinued due to lack of time in the curriculum and the increasing scope of the exploration phases.

PRESENT GEOPHYSICS CURRICULUM

In planning the courses of instruction in Exploration Geophysics, two pitfalls had to be avoided, which incidentally, are continuously faced by all technical schools. It was felt in 1926 and it is felt today, that a reasonable balance must be preserved between basic theory and principles on the one hand, and operation of instruments and field technique on the other. Over-emphasis of the first would tend to turn out men who were theorists with their feet off the ground as regards the objectives of a particular geophysical method applied as an exploration procedure; while too much time devoted to instrument operation, or too much field work, would lead to a trade school type of instruction which does not require the facilities of an institution of higher learning. The proper allocation of the time available for a given course, between theory and operating technique is a matter of opinion which will be discussed later.

The geophysical courses at the Colorado School of Mines are built around the fundamental engineering training, leading to the degree of Geological Engineer, with supplementary courses in higher mathematics and advanced physics.

Table 1 is a chart showing the program of work in the Geology-Geophysics option leading to the above named degree. The courses are grouped under the headings: Geophysics, Geology, Physics and Mechanics, Chemistry, Engineering, the Humanities, one including Military Training, Physical Education and Safety Work. Each course shows by a number-symbol, the required hours of lecture, laboratory, and preparation; the sum of these figures represents the total credit hours for each course. The chart shows further a partial list of fourth year courses which may be elective for seniors (E.F.S.), required of graduate students (R.G.S.), or elective for them (E.G.S.). A similar partial list of elective or required subjects for graduate students in Geology, Physics and Mechanics and Mathematics, is also shown. The chart gives the present status (academic year 1938–39) of the geophysics curriculum and pictures the expansion of instruction in this field since the first courses in 1927. It cannot of course, reflect the minor changes made from time to time, better to correlate our teach-

CHART OF COURSES COLORADO SCHOOL OF MINES GEOLOGY-GEOPHYSICS OPTION

GEOPHYSICAL ENGINEER

		GEOPHYSICS	GEOLOGY	PHYSICS & MECHANICS	MATHEMATICS	CHEMISTRY	ENGINEERING	THE HUMANITIES	MILITARY & PHYS. ED.
POST GRADUATE (Partly Open to Seniors)	Second Semester	Torsion Balance or Seismic Prosp. 3-3-3 (1 only for seniors) Geothermal Methods (E.G.S.) 2-1-3 G.P. Seminar 2-1-3 Grad. Research (R.G.S.)	Graduate Courses in Geology Such as: Geological Research Advanced Ore Deposits Micropaleontology Advanced Mineralography and Petrography Stratigraphic Research Advanced Petroleum Geology (3 or more, R.G.S.)	Potential Functions (E.F.S.) Advanced Theoretical Mechanics (E.G.S.) Vector Analysis (E.F.S.) Advanced Radio (E.G.S.) (2 or more, R.G.S.)	Higher Mathematics Such as: Fourier Series Theory of Functions Homography (E.F.S.) Empirical Equations Solid Analytic Geometry (E.F.S.) (3 or more, R.G.S.)				
	First Semester	Magnetic or Electrical Prosp. 3-3-3 (1 only for seniors) Radioactive Prosp. (E.G.S.) 2-1-3 G.P. Seminar 2-1-3 Grad. Research (R.G.S.)							
SENIOR	Second Semester	G.P. Investigations Max. 9 hrs. E.F.S. G.P. for Min. Engrs. E.F.S. in Min. 2-0-2	Sedimentology 2-0-2 Interpretation of Maps 0-6-0 Economic Geol. 3-0-2		Mathematics for Geologists and Geophysicists 3-0-6		Contracts and Specifications 2-0-2 Engineering Design (Mech. Engr.) 2-3-3	Principles of Economics 2-0-2	Advanced Military E.F.S.
	First Semester	Electronics 2-6-2	Petroleum Geol. 2-0-3 Ore Deposits 2-0-2 Optical Mineral. 0-6-1 Junior Geol. Inspection Trip Field Geol. 6 Wks. 20 Historical Geology 2-0-2 Structural and Engr. Geology 2-3-1 Elements of Paleontology 0-3-0	Thermodynamics 3-0-3		Note: 7 hours of electives each semester Sr. year Approved by Dept. head and Dean.		Principles of Economics 3-0-3 Engineering English 2-0-1	Advanced Military E.F.S.
JUNIOR	Second Semester	Applications of Geophysics 2-0-2		Principles of Electricity and Magnetism 3-3-3	Partial Differential Equations 3-0-5	Physical Chemistry 3-3-3 Note: 6 hrs. of electives each semester Jr. year			First Aid (U.S. Bureau of Mines) Advanced Military (Elective)
	First Semester	Review of Geophysical Methods 2-0-2	Petrology 0-6-0	General Physics 2-3-2	Ordinary Differential Equations 3-0-5	Physical Chem. 3-3-3	Strength of Materials 3-3-3 Kinematics of Machinery 2-3-2		Advanced Military (Elective) Military Camp
SOPHOMORE	2nd.Sem.		Mineralogy 0-6-0	General Physics 3-3-5 Mechanics 5-0-5	Integral Calculus 4-0-8		Graphic Statics 0-3-0 Mine Surveying 1-0-2	Technical Exposition 2-0-2	Physical Training 0-2-0 Military 1-2-0
	1st.Sem.		Mineralogy 0-6-0	General Physics 4-3-5	Differential Calculus 4-0-8	Quantitative Analysis 3-6-3	Mineral Land Surveying 1-0-2 Plane Surveying 20	Types of Literature 2-0-2	Physical Training 0-2-0 Military 1-2-0
FRESHMEN	2nd.Sem.		Introductory Historical 2-1-2		5-0-10	General and Qualitative 4-6-4	Desc.Geom & Engr. Dr'ng.1-5-2	English Composition 3-0-4	Physical Training 1-1-0 Military 1-2-0
	1st.Sem.		Elements 2-2-2		Algebra, Trigonometry, and Analytic Geometry 5-0-10	General Chemistry 4-3-5	Engr. Drawing 1-5-2	English Composition 4-0-4	Physical Training 0-2-0 Military 1-2-0
		GEOPHYSICS	GEOLOGY	PHYSICS AND MECHANICS	MATHEMATICS	CHEMISTRY	ENGINEERING: CIVIL, MECHANICAL, ENGINEERING DRAWING	THE HUMANITIES: LANGUAGES, ECONOMICS	MILITARY, PHYSICAL EDUCATION, SAFETY

Jan. 1939 (E.F.S.)= Elective for Seniors Course Symbol 2-3-1 for e.g. = 2hr.Lecture; 3hr.Lab.; 1hr.Preparation (R.G.S.) = Required for Graduate Students

ing with that of the other departments and to keep in step with accepted practices in mineral-industry engineering education.

MAIN FEATURES OF THE CURRICULUM

The plan of instruction for the first two years is virtually identical for all engineering options and will not be discussed further except to point that before taking *any* courses in geophysics, students have had a good ground work in geology, mathematics and physics. Two general courses in applied geophysics are required in the *Junior* year; the first deals with methods and principles and the second with applications to type geologic problems. The four major geophysical exploration procedures are covered in the one and the three main fields of application, (Petroleum Exploration, Ore Prospecting, and Civil Engineering applications) in the other. Of the four *graduate* courses, only one per semester is open to geophysics Seniors; because of the numerous other required courses in mathematics, physics, and geology that must be carried at the same time. Despite senior attendance, a graduate level can be maintained because the prerequisites are two semesters of general geophysics, ordinary and partial differential equations, and the other Junior courses. Graduate students must meet these prerequisites or are in some cases allowed to take some of them concurrently.

Each major graduate geophysical course consists of 3 lectures and one 3- to 4-hour laboratory period per week for the fifteen weeks of the semester, with field work on weekends or at the end of the semester. The lectures are arranged as follows with some minor variation depending on the method treated: I. Fundamental principles of the method; II. Physical properties of rocks and formations involved and methods for their determination; III. Instruments, their theory and operation; IV. Theory of subsurface effects and of interpretation; V. Presentation and discussion of results obtained in the field of oil exploration, mining, and civil engineering.

Laboratory work covers the measurement of rock properties, instrument calibration and testing and small-scale model experiments to illustrate principles applicable in a particular course. Field work is carried out at carefully selected sites where the geologic situation is known for the purpose of demonstrating the relationship between geologic conditions and geophysical indications. In the electrical and magnetic courses, surveys are made on geologic mining or placer problems; two weeks of Seismic reflection and torsion balance work

are carried out in Eastern Colorado at the end of the second semester.

The above arrangement represents a time distribution of 60 to 70% on fundamental theory and theory of interpretation; 10 to 15% on principles of operation and calibration of instruments; 5 to 10% on model experiments and rock property determinations, and 10 to 15% on field demonstration of the relationship of geology and geophysics.

In addition to the four major courses, six others of graduate standing and two undergraduate courses are offered: (1) Radioactive Prospecting; (2) Geothermal Investigations; (3–4) two semesters of Geophysical Seminar; (5–6) two graduate Research courses; (7) one general Geophysics course for Senior Mining Engineers; and (8) a course, "Geophysical Investigations," permits a limited amount of Senior student research. Two courses, one, Electronics and the other, "Mathematics for Geologists and Geophysicists" (offered by a member of the Mathematics Department who is on the Geophysics staff), complete the scope of our work. Altogether, 15 Geophysics courses are taught in the Department.

Requirements and program for candidates for the M.S. or D.Sc. degrees in Geophysics are more rigorous than those for the geological engineers degree with the geophysics major, especially in regard to higher mathematics, advanced physics, and advanced geology. The Geological Engineer degree requires a total of 480 credit hours; the Master's degree, a minimum of 120 credit hours of approved graduate work beyond the 4 year degree (of which the thesis can count only a maximum of 45 hours), and the Doctorate, a minimum of 120 credits above the Master's degree. As usual in educational institutions, credits and time lose their significance in this degree; the success with which the advanced program is followed and the character of the thesis submitted are the main issues.

PLANNING A GEOPHYSICS CURRICULUM AT AN ENGINEERING SCHOOL

Limitations were imposed in planning our program since it had to fit into the structure of an already established 4-year course of training engineers for the Mineral Industries. For most of our students, it is not economically feasible to add a fifth year to study geophysics; hence, seniors have been permitted to take a limited number of the advanced geophysics courses. Under this arrangement, certain admitted sacrifices in geology and other courses were necessary. It

was not possible to give a Geophysics major all the geology work taken by men graduating in that option *plus* the Geophysics courses with the necessary supplementary training in Mathematics and Physics. A similar situation would arise in building a curriculum for geophysics instruction within a department of Physics,[3] in which case sacrifices in Physics courses would be necessary to make room for the ground work in Geology. It is a moot question as to whether Geophysics training should be based on Geology first plus training in Physics and Mathematics, or on Physics first with supplementary work in Geology. We have chosen the Geology base at the Colorado School of Mines, although we are quite ready to admit that ours is not the *only* way. In developing prerequisite courses for the Geophysics program, the cooperation of other Departments was requested. The Mathematics Department instituted courses in advanced mathematics and the Physics Department arranged an advanced course in Physics for Juniors and a course in the fundamentals of Radio for Seniors. Courses in Vector Analysis, Potential Functions, and Advanced Theoretical Mechanics were established by the Mechanics Department.

A considerable investment in instruments and equipment was called for on the part of the administration in setting up the Department; it stands now at about $25,000 and is well balanced between the 4 major methods. As geophysical instruments have become somewhat more standardized, our needs have diminished along this line; further, we are under no competitive commercial pressure to have the very latest in equipment. Our purpose in the use of instruments (and this is a point I would like to emphasize) is to teach the *fundamentals* only, and to bring out to the student the relationships of Geology and Geophysics. It has never been the purpose of the Department to give instruction in *operating* geophysical instruments as an end in itself; the object is to familiarize students with them so that they can carry on a limited amount of field work for the above named purposes. In having the class in Seismic Prospecting run a short seismic reflection survey, it is felt that it gives them an appreciation of the general operation of commercial seismic work and interpretation problems arising in the field; the reactions of our graduates are quite definite on this matter.

It has never been a school policy to permit too great specialization of an engineer; since economic shifts may wipe out his means of

[3] See D. A. Keys, A.I.M.E. Round Table on Geophysical Education, (T.P. 950), Aug., 1938.

livelihood, our scheme of training follows the same thought; hence, our graduates have been known to enter related technical lines from the Geology-Geophysics option with considerable facility.

OTHER WORK OF THE GEOPHYSICS DEPARTMENT

The original statement of Mr. Ball called attention to the research aims of the Department of Geophysics. In the past 12 years, 23 advanced degrees have been granted in geophysics; of these, 13 were the Master's degree, and 10, the Doctorate. Each degree represents a research problem carried to a successful conclusion. The results of many of these researches have been made public in technical journals or in quarterlies of the Department; one of these appeared in July, 1932, and the other in January of 1937. It is planned to expand the scope of our research with the new quarters to be completed in September of this year; a considerable amount of new equipment will be installed; the building (a wing on the new Geology building of the School of Mines) will cost approximately $100,000.[4]

Dissemination of information on geophysics is a continuous activity with us. In addition to the two quarterlies noted, a third was published on geophysical methods in March, 1929, and a fourth, giving a list of geophysical references, in July, 1931. The Series of Publications of the Department, which included published articles by staff members and other men associated with the department and theses by graduate students, has reached the number 85. Preparation of abstracts for the Geophysical Section of the Annotated Bibliography of Economic Geology has been carried on by the Department since 1928.

DATA ON GEOPHYSICS STUDENTS TRAINED AT THE COLORADO SCHOOL OF MINES

In the period from January, 1927, to January 1, 1939, a total of 434 students have taken the two general courses in Geophysics. Of this number, 159 men have taken one or more of the graduate courses in addition. Fifty (or 31%) of these men had degrees from other institutions before entering the School of Mines, of which 15 came from foreign countries (Canada, Australia, New Zealand, Czechoslovakia, Belgium, Poland, Norway, Colombia, Venezuela, India, China and Japan). Some of these students were sent by their respective governments and some by foreign geophysical organizations; two of them

[4] C. A. Heiland; New Geophysics Laboratories at Mines, Mines Magazine, Vol. 29, No. 6, pp. 272–273 and 343, June, 1939.

were Commonwealth Fund Fellowship men. Fourteen of our own graduates, not included in the above 31%, returned for advanced work. The distribution of men among the different graduate courses is as follows: Magnetic Exploration, 104; Seismic Prospecting, 95; Electrical Prospecting, 80; Gravitational Work, 75; Radioactive and Geothermal methods, 20 and 15, respectively.

In January, 1938, a study was made of the positions occupied by 142 men who, up to that time, had taken one or more graduate courses

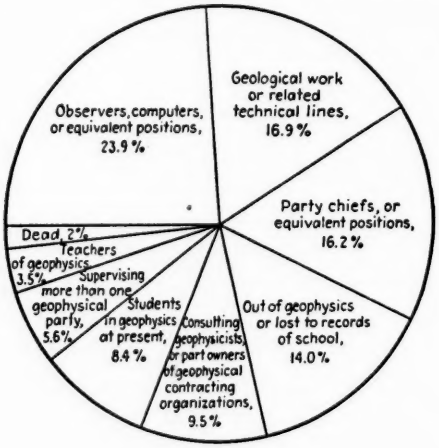
Geological work or related technical lines, 16.9%
Party chiefs, or equivalent positions, 16.2%
Out of geophysics or lost to records of school, 14.0%
Consulting geophysicists or part owners of geophysical contracting organizations, 9.5%
Students in geophysics at present, 8.4%
Supervising more than one geophysical party, 5.6%
Teachers of geophysics 3.5%
Dead, 2%
Observers, computers, or equivalent positions, 23.9%

FIG. 2.

in addition to the two general courses. The data assembled covered the 11-year period from January, 1927, and are shown in Fig. 2. The nearly equal percentages of party chiefs and supervisors which together total 21.8 per cent, tends to show that the training received equips men for supervisory technical positions as well as for the somewhat less highly paid posts. It is realized that there are always economic and individual situations involved where a man becomes a consulting geophysicist, but it is of interest that the training received is also apparently sufficient for such stations.

REPLIES OF GRADUATES TO INQUIRY CONCERNING COURSES

Geophysics graduates represented in regard to positions by the chart of Fig. 2 were asked in December, 1938, to give an opinion of their training. Their answers to questions concerning the distribution of time in a graduate geophysics course devoted to theoretical material, laboratory and field work are summarized in Table 2, which

suggests that the present 50 to 60 per cent time allotment to theoretical phases meets with the approval of the majority of graduates (53.2%); the minority is equally divided between an increase and a decrease. The present 10–15% time allotment to instrument operation and calibration is approved by 49.2% of the replies, while some 40% wish an increase. 58% consider the time now used for model

TABLE 2

TABULATION OF REPLIES FROM GEOPHYSICS-GRADUATES OF THE COLORADO SCHOOL OF MINES TO INQUIRY CONCERNING PREFERENCE IN THE DISTRIBUTION OF THEORETICAL, LABORATORY AND FIELD WORK

Theory of Interpretation / *Theory* of Instruments			Principles of *Operation* and Calibration of *Instruments*			*Model Experiments Rock-Property* Determinations			*Field Demonstrations* of Relationship between Geology and Geophysics		
less	more	as now (50–60%)	less	more	as now (10–15%)	less	more	as now (5–10%)	less	more	as now (10–15%)
23.4%	23.4%	53.2%	10.2%	40.6%	49.2%	6.7%	35.0%	58.3%	1.7%	62.8%	35.5%
(Per cents of total replies)											

experiments and rock-property determinations to be correct and 35% desire an increase. The most definite trend in the replies is that nearly 63% of the answers favor more field work for demonstrating the relations between geology and geophysics. It was my privilege to study the individual answers to this questionnaire; we were informed in no uncertain terms of our sins of omission and commission. Steps have been taken to improve our courses in view of the ideas expressed by our graduates just as they have been changed previously in response to suggestions from employers. In any changes, however, we have endeavored to maintain the original basis of fundamentals upon which these courses were founded.

CONCLUSION

It may be true that saturation levels have been reached in certain fields of commercial application of geophysics, yet undeniably much territory remains to be prospected even in the United States and particularly in foreign countries. The adaptation of Applied Geophysics to an ever increasing number of geologic and engineering problems holds promise; in the search for orebodies, geophysical work still has wide possibilities. Judging our field by its past development, one cannot help but be optimistic about its future course and the opportunities for men trained in the fundamentals and applications of Exploration Geophysics.

MEMORIAL

ERNEST EUGENE BLONDEAU

1904–1939

Ernest Eugene Blondeau died on December 5, 1939 from complications following an operation for acute appendicitis. Because of a particularly rugged constitution he responded magnificently after the operation and appeared to have the situation well in hand when the complications developed which ended his life.

He was born in St. Joseph, Missouri on August 27, 1904, and moved to Houston, Texas, at an early age. He attended Rice Institute and, majoring in physics and mathematics, obtained his Bachelor of Arts degree in 1926 and his Master of Arts degree in 1927. He was granted a Fellowship in Physics at Rice and was working on his Doctors degree in 1929 when he accepted employment with the Geophysical Research Corporation as chief of an electrical field party. His progress was rapid from that point and he occupied the position of technical advisor to this company at the time of his death.

He is survived by his widow, Maude Bryan Blondeau and three boys, Ernest Eugene Jr., Robert Wayne and John Pierre. His other survivors are his father, Mr. Eugene Pierre Blondeau of Goose Creek, Texas, and his sister, Mrs. W. H. Calkins of Houston, Texas.

Early in life Ernest suffered a severe attack of infantile paralysis which left him crippled. It was typical of his character that he neither allowed this affliction to retard his progress professionally nor did he permit it to interfere with his participation in outside activities. He was enthusiastic about sports and in recent years had made sailing a hobby.

He was endowed with an active mind and a fertile imagination and as a result made a number of contributions to his chosen field of geophysics. All phases of this subject interested him and he was constantly trying to improve the technique and interpretation involved in the different branches.

Blondeau's genial personality, pleasant disposition and sense of humor won him the sincere admiration of a large group of friends and his early death means a great loss to family, friends and business associates.

B. B. WEATHERBY

December 29, 1939.

REVIEWS

*Jahrbuch der Deutschen Mineraloelwirtschaft,** 1939/1940, prepared in collaboration with "Wirtschaftsgruppe Kraftstoff—Industrie" and "Fachgruppe Mineraloel" by Karl-Heinrich v. Thurmen, consultant on the Reichswirtschaftsministerium. Bound in full linen, 626 pp., RM 9.60, Published by "Naturkunde und Technik," Frankfurt a Main.

This volume the publishers announce as having grown beyond the scope of a yearbook into a compendium of the German Petroleum industry, although the title of yearbook is maintained.

The volume is divided into seven parts with the following captions: 1. General; 2. Organization-structure: 3. Legal foundations; 4. Petroleum-production, refining and distribution; 5. Science and Research; 6. Statistics; 7. Appendix. Parts 1 and 2 are likely to be of limited interest outside of Germany, except possibly to marketing interests. Part 3 is the most voluminous one and deals with the laws and regulations thereunder relating to the production, refining and marketing of crude oil and its products to taxes, import and export. Part 4 deals with the production of fuel and other oils from coal as well as petroleum. Part 5 is a tabulation of scientific organizations, those of the Nazi party receiving the most space. The A.P.I. is the only one in the U.S. mentioned on the single page devoted to foreign scientific organizations. Part 6 covers the statistics of production and consumption principally for Germany but there are some world statistics. The appendix includes a bibliography as well as a subject index.

In view of the extensive geophysical surveys which have been conducted in Germany one is astounded at the scant references to geophysical work. All reference to geophysical prospecting could be assembled on a single page with room to spare. The Deutsche Geophysikalische Gesellschaft is not even mentioned. Presumably it has not found a place in the far flung ramifications of Nazi party organization.

E. A. ECKHARDT

* Copy addressed to GEOPHYSICS received October 1939, Ed.

PATENTS

The following U. S. patents continue the quarterly listing initiated in the October issue. While not complete, the list represents a major portion of those pertinent to the art and of interest to exploration geophysicists. The very brief abstracts are presented merely as an indication of the nature of the material covered by the patents so that the reader may judge whether further investigation is warranted. The abstracts are not to be construed in any sense as an interpretation of the claims.

ELECTRICAL PROSPECTING

U.S. No. 2,172,271, L. F. Athy and H. R. Prescott, Iss. 9/5/39, Apl. 1/10/38 Assign. Continental Oil Co.

Method of Electrical Prospecting—A.C. of predetermined and preferably low frequency is passed through the earth between two separated electrodes. The resulting potential difference between two intermediate potential electrodes is recorded simultaneously with the input current. Response of the potential recorder is limited to the selected frequency by the use of suitable filters. Claims method permits relatively low input currents because of freedom from extraneous electrical interference.

U.S. No. 2,172,557, H. M. Evjen, Iss. 9/12/39, Apl. 3/30/37 Assign. Shell Development Co.

Electrical Method of Geophysical Exploration—A source of electric current (generally low frequency A.C.) having one terminal connected to the ground at one point and the other terminal connected to the ground at a plurality of points through a plurality of variable impedances . . . "Causing the current to flow through the ground in a plurality of non-crossing streams having different depths of penetration, equalizing the current in each stream by means of the variable impedances, varying the frequency of the current and determining the electrical properties of the ground at various depths by recording the change in the current intensity of each stream when the frequency of the current is varied."

U.S. No. 2,172,688, W. M. Barret, Iss. 9/12/39, Apl. 8/19/37 Assign. Engineering Research Corp.

Electrical Apparatus and Method for Geologic Studies—Radio frequency waves directed over the surface of the earth to locate anomalies. Receiver loop many wavelengths from transmitter avoids induction field and permits measurement of the departure of the radiation field from normal, such departures being related to subsurface geologic features.

U.S. No. 2,172,778, W. J. Taylor, Jr., Iss. 9/12/39, Apl. 3/1/37.

Method of and Apparatus for Geological Exploration—Suggests an improvement in electrical prospecting involving a special composite electrode or arrangement of electrodes comprising essentially multiple earth contacts encircling each other such as conconcentric ring-electrodes. In a two-terminal earthed system branch circuits supply current in the same direction to individual annular earth contacts and separate instruments in each branch circuit permit current measurements in the individual branches. It is claimed that the current stream in any particular branch circuit is confined to a region bounded by the current stream from adjacent electrode surfaces, thus permitting some control of the depth of current penetration from any particular branch circuit of the electrical quantity measured therein.

U.S. No. 2,174,343, J. J. Jakosky, Iss. 9/26/39, Apl. 5/24/37.

Method and Apparatus for Electrical Exploration of Subsurfaces—An A.C. or D.C. is passed through the earth between a pair of current electrodes while measurements are taken adjacent to one or both energizing electrodes, which measurements comprise magnetic field, or potential drops indicative of the angle of penetration of the mean path of the energizing current.

U.S. No. 2,177,346, Robert Saibara, Solomon Bilinsky and W. G. McLarry, Iss. 10/24/39, Apl. 4/18/38 Assign. E. E. Rosaire.

Exploration by Incremental Wave Distortion.—A modified Eltran method wherein the transient wave picked up by potential electrodes is passed through a distorting network which is adjusted until the resulting transient has a predetermined shape. The setting of the distorting network measures the received wave shape. Single transients or continuous waves (preferably square waves) may be used.

GRAVIMETRIC PROSPECTING

U.S. No. 2,179,892, A. R. Lindblad, Iss. 11/14/39, Apl. 11/24/37, Assign. Bolidens Gruvaktiebolag. Sweden.

Apparatus for Determining Gravity.—Temperature regulator for gravimeter. Instrument enclosed in metallic case of high thermal capacity with a bimetallic regulator and heating coil contacting the case, all supported in a thermos flask. Claims restricted to instrument in which capacity of a condenser varies with the force of gravity.

U.S. No. 2,182,298, Johan D. Malmqvist, Iss. 12/5/39, Apl. 2/8/38 Assign. Bolidens Gruvaktiebolag, Sweden.

Method of Measuring Gravity—Comprises a measurement of the deviation of a plumb line. A telescope and scale at a distance from a plumb line measures the change in vertical angle of a mirror attached to the plumb line when the telescope is held fixed and the plumb line and mirror apparatus is moved to various points along a profile.

U.S. No. 2,183,115, F. G. Boucher, Iss. 12/12/39, Apl. 6/20/36 Assign. Standard Oil Development Co.

Multiple Gravity Meter—A null-reading type instrument comprising essentially a horizontal spring under tension supporting a centrally located weight system having one vertical and one horizontal arm. The torque due to the horizontal weight is balanced by torsion applied at opposite ends of the spring. Changes in gravity affect the position of the weight system which is restored to its original position by a very weak auxiliary spring. The presence of the vertical arm and adjustment of its center of mass with respect to the axis of the system obviously permits adjustment of the period (or sensitivity) up to the point of instability. The requirement for very accurate levelling of the instrument is met by taking several readings in different azimuths, or alternatively several separate instruments oriented in different azimuths on a single base are used and their readings averaged.

SEISMOGRAPH PROSPECTING

U.S. No. 2,180,949, L. W. Blau, W. D. Mounce and W. M. Rust, Jr., Iss. 11/21/39, Apl. 7/11/36 Assign. Standard Oil Development Co.

Feedover Device—Seismograph channel mixing using electrical networks within and between channels. Networks comprise filter sections. Coupling between channels ac-

complishes an increase in the ratio of desired to undesired vibrations recorded. Coupling between channels may be provided by closely spacing transformers or filter components in adjacent amplifier channels.

WELL LOGGING

U.S. No. 2,170,857, R. D. Elliott, Iss. 8/29/39, Apl. 12/1/37 Assign. Schlumberger Well Surveying Corp.

Recording Apparatus and Method—Recorder for well logging. Effects on current and potential electrodes separately recorded. Camera electrically driven in synchronism with electrode motion through the well bore. Spots from current and potential electrode galvanometers are focused at points separated lengthwise of film so that effective elevations of the two resistivity measurements coincide on the film. Contactor in camera drive flashes depth marks on the film.

U.S. No. 2,172,625, Conrad Schlumberger, Iss. 9/12/39, Apl. 6/28/35 Assign. Societe de Prospection Electrique Procedes Schlumberger, France.

Process for Investigating Permeable Strata Tranversed by a Boring—A process for investigation of the permeability of the strata traversed by a drill hole consisting of effecting at different depths a series of measurements of a predetermined physical parameter of the strata, which parameter (i.e.—electrical resistivity) being a function of the liquid penetrating the strata. Then effecting a second measurement of this parameter after elapsed time or added pressure has permitted greater penetration of the fluid in the well bore, or alternatively by changing the nature of the fluid in the bore. Comparison of the two series of measurements permits the location of permeable strata where penetration of the liquid has modified the value of the physical parameter or impervious strata where no change has occurred.

U.S. No. 2,174,638, Conrad Schlumberger, Iss. 10/3/39, Apl. 8/31/35 Assign. Societe de Prospection Electrique Procedes Schlumberger, France.

Method and Apparatus for Electrical Survey of the Formations Cut by a Bore Hole—Measurement of resistivity and self-potential with the same exploring electrodes is accomplished by interrupting energizing current at intervals with a rotary switch which synchronously switches measuring electrodes from resistivity galvanometer to self-potential galvanometer.

U.S. No. 2,176,169, H. G. Doll, Iss. 10/17/39, Apl. 7/23/36 Assign. Societe de Prospection Electrique Procedes Schlumberger.

Method and Arrangement for Determining the Direction and the Value of the Dip of Beds Cut by a Bore Hole—Dip of strata in wells is electrically measured by using multiple spaced electrodes or a rotating electrode to determine variations of self-potential across the bore hole.

U.S. No. 2,179,593, J. J. Jakosky, Iss. 11/14/39, Apl. 9/28/36 Assign. Schlumberger Well Surveying Corp.

Electrical Method and Apparatus for Determining the Characteristics of a Geological Formation Traversed by a Bore Hole—Electro-chemical potential for well logging. Measurement is made of variations in the relative potentials between an exploring electrode in the well and surface electrodes spaced from the top of the well. Natural or artificial earth currents cause the electrochemical effect which it is suggested can be measured inside of casing.

U.S. No. 2,181,601, J. J. Jakosky, Iss. 11/28/39, Apl. 12/21/38.

Method and Apparatus for Continuous Exploration of Bore Holes—Describes a continuous electrical logging device using the drill bit, and including the entire drill stem or cable, as one current electrode and an electrode remote from the well as the other. Potential differences are measured between two electrodes at the surface located at such points that the potential difference varies when the energizing current takes different paths between the current electrodes. The remote current electrode is sufficiently removed from the well so that changes in the electrical characteristics of the formation adjacent to the drill bit will produce variations in the potential measurements. Continuous recording while drilling is disclosed with commutating means alternately to measure normal earth current fluctuations and eliminate them from the record.

THE SOCIETY ROUND TABLE

ANNOUNCEMENT OF ANNUAL MEETING—1940

The Annual Meeting of the Society of Exploration Geophysicists will be held at the Stevens Hotel, Chicago, Ill., April 9 to 11 inclusive.

Tuesday, April 9—A.M.—Business Meeting followed by technical session.
P.M.—Technical Session.

Wednesday, April 10—A.M.—Joint Session with the A.A.P.G. when President Eckhardt, and President Ley of the A.A.P.G. will address both Societies.
P.M.—Technical Session.

Thursday, April 11—A.M.—Technical Session.
P.M.—Technical Session.

The A.A.P.G. Meetings, in Chicago, are scheduled for April 10 to 12 inclusive.

The above schedule is tentative and further details concerning the program will be mailed to members several weeks before the meeting.

ANNUAL MEETING

PROGRAM COMMITTEE

Chairman—W. T. Born, P. O. Box 2040, Tulsa, Oklahoma

J. H. Crowell, 2011 Esperson Bldg., Houston, Texas

E. E. Rosaire, 321 Esperson Bldg., Houston, Texas

The committee will contact as many members as possible with requests for papers. Obviously it is impossible to solicit each member of the Society individually so if you have a paper to contribute please send it in regardless of whether you have received a personal invitation to do so.

The committee is especially interested in obtaining papers from members in foreign countries and hopes that such papers will be submitted to be read by proxy or by title.

Members desiring to submit papers will greatly aid the Program Committee if they will send copy to any one of the above prior to March 20.

To S. E. G. Members:

In the October issue of Geophysics some plans were reported for the annual meeting of 1940. In view of the change in those plans as indicated in the above Announcement I feel that an explanation should be made.

Early in December your president found the opportunity to spend a few days in Houston, enroute east from California and was privileged to visit with four past presidents of the Society, and through them became aware of a strong sentiment to cancel our arrangements for a separate meeting which tentatively had been scheduled to be held in Dallas beginning April 15th and therefore immediately following the A.A.P.G. meeting in Chicago. It was the consensus of opinion of these representatives of your Society that we should meet with the geologists in Chicago despite the admitted dis-

advantages of such a plan. Since several of these men had previously favored a separate meeting I believed it desirable to call this development to the attention of the executive committee and did so by letter requesting a reconsideration of the decision previously made. Several members of the executive committee consulted with members who could be reached conveniently and on the basis of their findings changed their vote.

We have therefore now decided to meet with the A.A.P.G. in Chicago. The executive committee as so decided because a substantial majority of the members of the Society consulted favored meeting in Chicago. Several members of the executive committee stated that they were voting against their own personal convictions because of the sentiment revealed by their canvasses of members. They and your president feel that the time and place of S.E.G. meetings should be so chosen as to insure a maximum attendance of its own members and that subject to this consideration we should meet as frequently as possible jointly with the A.A.P.G.

I am pleased that we shall after all meet jointly with the geologists in Chicago. At the same time I must confess some uncertainty as to the wisdom of the decision we have made. It is possible, and perhaps likely, that the members consulted by your executive committee are predominantly those who would be least deterred from attending meetings by considerations of time and expense. A vote on the question in Chicago would be equally inconclusive because it would be representative only of those members who found it practicable to go to Chicago. I hope that the executive committee for next year will have a better basis for deciding upon a policy for the times and places at which meetings of the Society shall be held.

January 8, 1940. E. A. ECKHARDT, *President*

MEMBERSHIP APPLICATIONS APPROVED FOR PUBLICATION

The Executive Committee has approved for publication the names of the following candidates for membership in the Society. This publication does not constitute an election, but places the names before the membership at large. If any member has information bearing on the qualifications of these nominees, he should send it to the Secretary within thirty days. (Names of sponsors are placed beneath the name of each nominee.)

ACTIVE

Robert Barling
Craig Ferris, E. S. Sherar, L. M. Mott-Smith

Gerald Roy Brotherhood
(Article III-C-1 of the Constitution)

Leonard Irving Brown
D. C. Blevins, S. C. Stoneham, J. E. Jonsson

Paul Frederick Clement
Leo J. Peters, O. F. Ritzman, E. A. Eckhardt

Richard Harry Dana
Sidon Harris, J. R. Gilliland, R. Clare Coffin

Malcolm Osborn Gibson
F. Goldstone, D. S. Hughes, R. D. Miller

Martin James Gould
Henry Salvatori, Joseph A. Sharpe, Neil R. Sparks

George Carl Howard
S. Zimmerman, A. B. Bryan, R. W. Gemmer

Bernard George Hubner, Jr.
George C. McGhee, J. H. Pernell, John A. Gillin
John Allan Legge, Jr.
K. H. Crandall, J. W. Hoover (Article III-C-1 of the Constitution)
Hans Lundberg
C. A. Heiland, Louis B. Slichter, E. G. Leonardon
Louis W. Minturn
W. W. Waring, T. I. Harkins, A. L. Smith
Ernest Turnley Nichols
C. Lundy, Harris Cox, Phil P. Gaby
John Edmond Palmer
R. Davies (Article III-C-1 of the Constitution)
Ernest Rowland Selby
M. A. Boccalery, Henry Salvatori, Dean Walling
Russell Edward Shafer
L. G. Ellis, John F. Anderson, W. E. Hollingsworth
Leslie C. Spencer
W. B. Hogg, Edward W. Fisher, L. I. Freeman
Albert William Taylor
W. T. Born, L. Y. Faust, Andrew Gilmour
Norman Pascal Teague
A. L. Selig, R. H. Ray, J. C. Pollard
Willard Harmond Tracy
Walter A. English, Hugo Benioff, A. B. Nomann
Gardner Pond Wilson
L. M. Mott-Smith (Article III-C-1 of the Constitution)

ASSOCIATE

Robert Beresford
L. M. Mott-Smith (Article III-C-1 of the Constitution)
Murray Buckner Flournoy
Andrew Gilmour, John L. Ferguson, Frank W. Borman
George Esmond Higgins
(Article III-C-1 of the Constitution)
Joseph Vincent McGinity
W. T. Born, J. E. Owen, J. M. Kendall
Louis J. Paddison
L. B. Slichter, Sidon Harris, R. Clare Coffin
James Eugene Stones
R. B. Moran, Jr., M. A. Boccalery, John W. Mathews
George Andrew Sullivant
D. C. Blevins, III, S. C. Stoneham, J. E. Jonsson
George Wheaton
M. C. Born, Phil P. Gaby, M. A. Boccalery

TRANSFER FROM ASSOCIATE TO ACTIVE STANDING
(Executive Order Number 18)

William M. Wells
Rex H. White

The ballot reprinted below was mailed all active members of the Society of Exploration Geophysicists on January 12, 1940. Active members who failed to receive ballots may secure them by notifying J. F. Gallie, Business Manager, P.O. Box 777, Austin, Texas. *Only active members who have paid 1940 dues are eligible to vote.*

OFFICIAL BALLOT

THE THE YEAR MARCH, 1940—MARCH, 1941
For the Election of Officers for the
SOCIETY OF EXPLORATION GEOPHYSICISTS

FOR PRESIDENT AND VICE-PRESIDENT

Vote Once for President (First Choice) and Once for Vice-President

	FIRST CHOICE (FOR PRESIDENT)	SECOND CHOICE (FOR VICE-PRESIDENT)
W. T. BORN	☐	☐
H. C. CORTES	☐	☐
H. B. PEACOCK	☐	☐
_____	☐	☐

FOR SECRETARY-TREASURER

VOTE ONCE

A. GILMOUR	☐
H. HOOVER, JR.	☐
_____	☐

FOR EDITOR

VOTE ONCE

W. M. RUST, JR.	☐
R. D. WYCKOFF	☐
_____	☐

After voting, place in accompanying official envelope, which should then be signed and mailed to the

SECRETARY-TREASURER,

J. H. Crowell
3416 Ella Lee Lane
Houston, Texas

PERSONAL ITEMS

Roy Lee Gallaway of the Texas Company is now in Egypt. He may be addressed at 3 Sharia Birgas, Garden City, Cairo, Egypt.

Kenneth K. Kennedy of the Carter Oil Company is doing seismograph work in New Zealand. He may be addressed at Box 108, New Plymouth, New Zealand.

Charles F. Reed, Jr., is acting as Station Manager for Dowell, Incorporated, in Bakersfield, California, where he may be addressed at Box 1402.

George B. Stone, formerly of the Independent Exploration Company, is now serving as a geologist for the Adams Oil and Gas Company, 2305 Esperson Building, Houston, Texas.

Robert Urick has left the California Institute of Technology to accept a position with the Petty Geophysical Engineering Company, Box 2061, San Antonio, Texas.

Richard H. Woodward of the Schlumberger Well Surveying Corporation has moved from El Dorado, Arkansas, to Jackson, Mississippi, where he may be addressed at Box 131.

M. H. Jameson of Seismograph Service, Incorporated, has returned from work in South America. He is at present located at 12 Kay Terrace, Rochester, New York.

Daniel F. Elam, District Engineer for International Geophysics, Incorporated, is now located in Mount Vernon, Illinois. He may be reached at Box 179 in that city.

Ernest W. Lemke of the Carter Oil Company has moved from Maryville, Missouri, to Aberdeen, Mississippi, where he may be addressed at Box 423.

E. A. Fain of Geophysical Service, Incorporated, may be addressed c/o Indian Oil Concessions, Limited, 2 Bath Island Road, Karachi, India.

John J. Rupnik has left the Texas Company to accept an appointment as a graduate assistant at the California Institute of Technology. His mailing address is 364 South Hudson Avenue, Pasadena, California.

K. E. White has been transferred from Holland to the Standard Oil Company of Egypt, 22, Sharia Kasr el Nil, Cairo, Egypt.

Marvin Romberg of the Humble Company has returned to the Netherlands East Indies. He may be addressed c/o R. L. Suggs, Humble Seismograph Crew, Geological Department, N. K. P. M., Soengei Gerong, Palembang, Sumatra.

Francis A. Roberts has returned to the States and may be addressed at 1732 Elm Street, Lake Charles, Louisiana.

George B. Somers is working for the Socony-Vacuum Oil Company, Sharia Ibrahim Pasha, Cairo, Egypt, having left the United States on November 25th of last year.

Stanley W. Wilcox of the Seismograph Service Corporation has transferred from Mattoon, Illinois, to Box 111, Flora, Illinois.

L. F. Fischer of the Phillips Petroleum Company is now located in Floydada, Texas, where he may be addressed at Box 605.

J. W. de Bruyn has left the Netherlands East Indies to work for the Burmah Shell Oil Company, Limited, Karachi, India.

T. J. Etherington of the Richmond Petroleum Company of Colombia has moved from Ibague to Bogota. He may be addressed care of that company, Apartado Nacional 2760, Apartado Aereo 3631, Bogota, Colombia, South America.

L. C. Paslay of the National Geophysical Company is now located in Larchmont, New York, at 74 Edgewood Street.

Basil B. Zavoico, geologist for the Chase National Bank, has moved his offices from Houston, Texas, to 18 Pine Street, New York, New York.

B. D. Lee has moved from Barranquilla, Colombia, to Caracas, Venezuela, where he may be reached through the Texas Company of Venezuela, Limited, Apartado 267.

Neal Clayton has returned from Venezuela to the States, and may be addressed at Box 132, Ripley, Mississippi.

The Mott-Smith Corporation has moved its offices from the 15th to the 9th floor of the Shell Building in Houston, Texas.

E. L. De Golyer, honorary member of the Society, has been appointed to a distinguished professorship at the University of Texas in Austin. Dr. De Golyer will take up his new duties February 1st, at the start of the second semester.

Ralph C. Loring is now working in South America for the International Petroleum Company, Limited. He may be addressed at Apartado 803, Guayaquil, Ecuador.

Charles C. Lash of the Western Geophysical Company has moved from Los Banos, California, to 3961 Halldale Avenue, Los Angeles, California.

Marion J. Moore is now geologist for the Transwestern Oil Company, Box 88, San Antonio, Texas.

STATEMENT OF THE OWNERSHIP, MANAGEMENT, CIRCULATION, ETC., REQUIRED BY THE ACTS OF CONGRESS OF AUGUST 24, 1912, AND MARCH 3, 1933

of GEOPHYSICS, published quarterly at Menasha, Wisconsin, for October 1939.

State of Texas } ss.
County of Travis }

Before me, a Notary Public in and for the State and county aforesaid, personally appeared J. F. Gallie, who, having been duly sworn according to law, deposes and says that he is the Business Manager of the Geophysics and that the following is, to the best of his knowledge and belief, a true statement of the ownership, management (and if a daily paper, the circulation), etc., of the aforesaid publication for the date shown in the above caption, required by the Act of August 24, 1912, as amended by the Act of March 3, 1933, embodied in section 537. Postal Laws and Regulations, printed on the reverse of this form, to wit:

1. That the names and addresses of the publisher, editor, managing editor, and business managers are: Publisher, George Banta Publishing Company, Menasha, Wisconsin; Editor, R. D. Wyckoff, Box 2038, Pittsburgh, Pennsylvania; Managing Editor, R. D. Wyckoff, Box 2038, Pittsburgh, Pennsylvania; Business Manager, J. F. Gallie, Box 777, Austin, Texas.

2. That the owner is: (If owned by a corporation, its name and address must be stated and also immediately thereunder the names and addresses of stockholders owning or holding one per cent or more of total amount of stock. If not owned by a corporation, the names and addresses of the individual owners must be given. If owned by a firm, company, or other unincorporated concern its name and address, as well as those of each individual member, must be given.) Owned by The Society of Exploration Geophysicists, Executive Committee: R. D. Wyckoff, Editor, Box 2038, Pittsburgh, Pa.; J. F. Gallie, Business Manager, Box 777, Austin, Texas; E. A. Eckhardt, President, Box 2038, Pittsburgh, Pa.; W. T. Born, Vice-President; Box 2040, Tulsa, Oklahoma; F. M. Kannenstine, Past President, 2011 Esperson Bldg., Houston, Texas.

3. That the known bondholders, mortgagees, and other security holders owning or holding 1 per cent or more of total amount of bonds, mortgages, or other securities are: (If there are none, so state.) None; non-profit organization.

4. That the two paragraphs next above, giving the names of the owners, stockholders, and security holders, if any, contain not only the list of stockholders and security holders as they appear upon the books of the company but also, in cases where the stockholder or security holder appears upon the books of the company as trustee or in any other fiduciary relation, the name of the person or corporation for whom such trustee is acting, is given; also that the said two paragraphs contain statements embracing affiant's full knowledge and belief as to the circumstances and conditions under which stockholders and security holders who do not appear upon the books of the company as trustees, hold stock and securities in a capacity other than that of a bona fide owner; and this affiant has no reason to believe that any other person, association, or corporation has any interest direct or indirect in the said stock, bonds, or other securities than as so stated by him.

5. That the average number of copies of each issue of this publication sold or distributed, through the mails or otherwise, to paid subscribers during the twelve months preceding the date shown above is not daily. (This information is required from daily publications only.)

J. F. GALLIE

Sworn to and subscribed before me this 27th day of September 1939.

[SEAL] ANNE W. GREEN

(My commission expires June 1, 1941.)